Solutions to Integration Problems

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1. Perform integration by parts with $u = x, dv = \sec^2 x dx$. The answer is $x \tan x + \ln|\cos x| + C$
2. Perform the substitution $u = 2x + 1$. The answer is $2x + \ln|2x + 1| + C$
3. Perform the substitution $u = x^2$. The answer is $\frac{7}{2}e^{x^2} + C$
4. Use the fact that $x^2 + 6x + 9 = (x + 3)^2$. The answer is $-\frac{1}{x+3} + C$
5. Since $\sin(x^3)$ is an odd function and the interval is symmetric around 0, the integral is zero. (Note that we would not want to find an antiderivative for this function.) 0
6. Perform the substitution $u = 3x + 4$. The answer is $\frac{2}{9}(31\sqrt{31} - 7\sqrt{7})$
7. Integrate by parts with $u = \tan^{-1} x$ and $dv = x$. Complete the integration by using long division. The answer is $-\frac{x}{2} + \frac{\tan^{-1} x}{2} + \frac{1}{2}x^2 \tan^{-1} x + C$.
8. Integrate by parts twice and note that we obtain the original integrand. The answer is $\frac{1}{2}e^x(\sin x - \cos x) + C$
9. Integrate by parts with $u = \ln x, dv = xdx$. The answer is $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$
10. Use the trigonometric identity $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$. The answer is $\frac{1}{2}x - \frac{1}{28}\sin(14x) + C$
11. Perform the substitution $u = x^4 + x^3 + x^2 + x + 1$. The answer is $\ln|x^4 + x^3 + x^2 + x + 1| + C$
12. Use partial fractions. The answer is $\frac{9}{2}\ln|x - 1| - 15\ln|x - 2| + \frac{21}{2}\ln|x - 3| + C$
13. $\frac{1}{2}\sin(x^2) + C$. Substitute for x^2.
14. $x^2 \sin x + 2x \cos x - 2\sin x + C$. Integrate by parts twice.
15. $\frac{1}{2}\ln|x - 1| + \frac{3}{2}ln|5 + x| + C$. Integrate by parts after factorizing the denominator.
16. $\frac{1}{2}x + \frac{1}{4}\sin(2x) + C$. Use trig identity $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$
17. $\frac{1}{2}x^2 \sin(x^2) + \frac{1}{2}\cos(x^2) + C$. Substitute for x^2, and then use parts - or just use parts with $u = x^2$ and $dv = x\cos(x^2)$.
18. $\sin(x) - \frac{1}{3}\sin^3(x) + C$. Use trig identity $\cos^2 x = 1 - \sin^2 x$ by rewriting integrand as $\cos^2(x)\cos(x)$. Substitute for $\sin^2 x$.
19. 1. Integrate by parts.
20. $x(\ln x)^2 - 2x\ln x + 2x + C$. Integrate by parts twice.
21. $\frac{1}{3}(\ln x)^3 + C$. Substitute for $\ln x$
22. $\ln 3$. Substitute $u = \ln x$.
23. $\frac{1}{2}(x^2 + 1)\ln(x^2 + 1) - \frac{1}{2}(x^2 + 1) + C$. Substitute for x^2 and then do by parts. Your answer may differ by a constant, of course.
24. $x - 2\arctan x + C$. Realize that the fraction equals $1 - \frac{2}{x^2+1}$. You can also use long division to do this.
25. $x - \ln|x + 1| + \ln|x - 1| + C$. Note $1 + \frac{2}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}$. Note that long division could be used here as well.
26. $\frac{1}{2}\tan^{-1}(x^2) + C$.
27. $\sqrt{x^2 + 1} + C$. Substitute $u = x^2 + 1$.
28. $\frac{1}{4}\ln|e^x - 2| - \frac{1}{4}\ln|e^x + 2| + C$. Substitute for e^x followed by partial fractions
29. $\frac{1}{2}\ln|e^{2x} - 4| + C$. Substitute for e^{2x}.
30. $2\sqrt{7} - 2\sqrt{5}$. Substitute for $e^x + 4$.
31. $2\ln|x - 3| - \ln|x + 1| + C$. Partial fractions.
32. $x + 2\tan^{-1} x + C$. Realize that the fraction equals $1 + \frac{2}{x^2+1}$.
33. $\frac{1}{3}e^{x^3} + C$. Substitute for x^3.
34. $-\frac{3}{2}\ln|\cos 2x| + C$. Write as $\frac{\sin x}{\cos x}$ and then substitute for $\cos x$.
35. $e^{\tan x} + C$. Substitute for $\tan x$.
36. $-\frac{\cos(2\sin x)}{2} + C$. Substitute for $\sin x$.

37. $\frac{\ln(1+x)}{(1+x)} + \frac{1}{(1+x)} + C$. Let $u = x+1$ and then integrate by parts.

38. $-\frac{2}{1+x} + C$. Factor the denominator and look at the problem.

39. $\frac{1}{2}\ln\frac{x}{x+2} + C$. Partial Fractions.

40. $\frac{3}{4}\ln|x-1| - \frac{1}{2(x-1)} + \frac{1}{4}\ln|x+1| + C$. Partial fractions.

41. $3x\sin x + 3\cos x + C$. Integrate by parts.

42. $\frac{1}{3}(x^2+2x)^{\frac{3}{2}} + C$. Write as $(x+1)\sqrt{x^2+2x}$. Substitute for x^2+2x.

43. $1 - \cos(1)$. Substitute for $\ln x$.

44. $\frac{x}{2}\big(\sin(\ln(x)) - \cos(\ln(x))\big) + C$. Integrate by parts twice, look to switch terms over.

45. $\frac{-1}{3x} - \frac{\arctan(x/3)}{9}$. Partial Fractions.

46. $\frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + C$. Write fraction as $\frac{x(x^2+1)-x}{x^2+1}$. Split the fraction and split the integral across the minus sign. Substitute for x^2 on the right side. Another way would to be approach via long division.

47. $\ln|x| - \frac{1}{2}\ln|x^2+1| + C$. Partial Fractions.

48. $\frac{1}{3}\ln\left|\frac{1-\cos(x)}{4-\cos(x)}\right| + C$. Substitute for $\cos x$. Partial fractions.

49. $2e^{\sqrt{x}}$. Substitute for $\sqrt{x}$.

50. $2e^{-x}(-x-1)$. Integrate by parts with $u = 2x$ and $dv = e^{-x}$.

51. $\frac{1}{2}\ln(x^2+1) + \tan^{-1}(x)$. Write the fraction as $\frac{x}{x^2+1} + \frac{1}{x^2+1}$.

52. $e^{(x+1)^2}$. Write as $(2x+2)e^{(x+1)^2}$. Substitute for $(x+1)^2$.

53. $\frac{2}{x} - \ln|x-1| + \ln|x|$. Integrate by partial fractions. Remember that the x^2 in the denominator adds both an x^2 and an x factor to the partial fraction method.

54. $-\sqrt{1-x^2}$. Substitute $u = 1-x^2$.

55. $-\frac{1}{2}\cos(x^2-1)$. Substitute for x^2-1.

56. $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$. Factor $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$. Obtain this through long division.

57. $\frac{(x^2+1)^5}{5} + \frac{2(x^2+1)^3}{3} + x^2 + 1$. Substitute $u = x^2+1$.

58. 0. Realize $\frac{x^3}{\sqrt{1-x^2}}$ is an odd function.

59. $2e^3/9 + 1/9$. Integrate by parts

60. $-\frac{1}{4}\cos(2x) + C$. Substitute $u = \cos(x)$.

61. $\ln\ln\ln x + C$. Substitute $u = \ln(x)$.

62. $-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3(2x) + C$ Substitute $u = 2x$. Remember $\sin^2(u) = 1 - \cos^2(u)$.

63. $x\ln\sqrt{x} - x/2 + C$ Remember $\ln(x^a) = a\ln(x)$. Integrate by parts.

64. $\frac{3}{4}\ln|x-2| + \frac{1}{4}\ln|x+2| + C$. Partial Fractions.

65. $e^{e^x} + C$. Note that $e^{e^x+x} = e^{e^x}e^x$. Substitute $u = e^x$.

66. $\ln(1+\sin(x))$. Substitute $u = 1+\sin(x)$.

67. $\frac{x^2}{2} + \ln|x-1|$. You can simplify the integrand to $x + \frac{1}{x-1}$ using long division followed by factoring.

68. $\frac{\ln|x|}{a^2} - \frac{\ln|x^2+a^2|}{2a^2}$. Long division followed by Partial Fractions.

69. $-(x^2-a^2)^{-0.5} + C$. Substitute $u = x^2 - a^2$

70. $\frac{x^2(\ln x)^2}{2} - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$. Integrate by parts twice.

71. $3\left(\ln 2 - \frac{1}{2}\right)$. Substitute $u = x^{\frac{1}{3}}$. Then $u^3 = x$ so $3u^2du = dx$.

72. $\tan x - x + C$. Use Trig. Identity $\tan^2(u) = \sec^2(u) - 1$.

73. $\frac{\pi}{12}$. Note that the integrand can be written as $\frac{1}{e^x + \frac{1}{e^x}}$ which in turn equals $\frac{e^x}{e^{2x}+1}$. Now Substitute $u = e^x$. Look for the integral of ArcTan.

74. $-\frac{1}{36}$. Integrate by parts.

75. $\frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{e}}\right)$. Substitute $u = -\frac{1}{x}$.

76. $\sqrt{x^2+3} + C$. Substitute $u = x^2+3$.

77. $x^2/2 + C$. Realize $e^{\ln x} = x$.

78. $1/2\ln(x^2+1) - \ln|x+1| + C$. Partial Fractions.

79. $t\ln t - t + C$, where $t = \sin x$. Substitute $u = \sin x$. Then integrate by parts.

80. $2(t - \arctan(t)) + C$, where $t = \sqrt{x}$. Substitute $u = \sqrt{x}$ so $u^2 = x$ and $2udu = dx$.

81. $(2/3)t^{3/2} + C$, where $t = \ln x$. Substitute $u = \ln x$.

82. $(x^2 - 2x + 2)e^x + C$. Integrate by parts twice.

83. $x\ln(x^2+1) - 2x + 2\arctan(x) + C$. Integrate by parts.

84. $t\ln t - t + C$, where $t = \ln x$. Substitute $u = \ln x$. Then by parts.