Section 7.4

② a) $\sum_{k=1}^{4} k \sin \frac{k\pi}{2}$

$= (1)\sin\frac{(1)\pi}{2} + \cdots + (4)\sin\frac{(4)\pi}{2}$

$= 1 + 0 + -3 + 0 = \boxed{-2}$

b) $\sum_{j=0}^{5} (-1)^j$

$= (-1)^0 + \cdots + (-1)^5$

$= 1 - 1 + 1 - 1 + 1 - 1 = \boxed{0}$

d) $\sum_{m=3}^{5} 2^{m+1}$

$= 2^{(3)+1} + \cdots + 2^{(5)+1}$

$= 2^4 + 2^5 + 2^6 = \boxed{112}$

⑥ $1 + 2 + 2^2 + 2^3 + 2^4 = \boxed{\sum_{k=0}^{4} 2^k}$

$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4 =$

⑨ $1 - 3 + 5 - 7 + 9 - 11$

$\sum_{k=1}^{6}(2k-1) = 1 + 3 + 5 + 7 + 9 + 11$

$\sum_{k=1}^{6}(-1)^{k+1} = 1 - 1 + 1 - 1 + 1 - 1$

combine: (multiply) $\boxed{\sum_{k=1}^{6} (-1)^{k+1}(2k-1)}$

Section 7.4

(16) $$\sum_{k=3}^{100} k = \sum_{k=1}^{100} k - \sum_{k=1}^{2} k = \underbrace{\frac{1}{2}(100)(100+1)}_{\text{Theorem 7.4.2}} - (1+2) = \boxed{5047}$$

(44) $$\sum_{k=1}^{50}\left(\frac{1}{k} - \frac{1}{k+1}\right) = \underbrace{\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \cdots + \left(\frac{1}{50}-\frac{1}{51}\right)}_{\text{Telescoping Series}} = \boxed{\frac{50}{51}}$$

Section 11.3

(1) a) $2 + \frac{2}{5} + \frac{2}{5^2} + \cdots + \frac{2}{5^{k-1}} + \cdots$

$S_1 = 2$

$S_2 = 2 + \frac{2}{5} = \frac{12}{5}$

$S_3 = 2 + \frac{2}{5} + \frac{2}{5^2} = \frac{62}{25}$

$S_4 = 2 + \frac{2}{5} + \frac{2}{5^2} + \frac{2}{5^3} = \frac{312}{125}$

$$S_n = \frac{a - ar^n}{(1-r)} = \frac{2 - 2\left(\frac{1}{5}\right)^n}{\left(1-\frac{1}{5}\right)} = \boxed{\frac{5}{2} - \frac{5}{2}\left(1/5\right)^n}$$

$$\lim_{n\to\infty} S_n = \frac{a}{(1-r)} = \frac{2}{1-\frac{1}{5}} = \frac{5}{2}, \quad \underline{\text{converges}}$$

Section 11.3

① (con't)

b) $\frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} + \cdots + \frac{2^{k-1}}{4} + \cdots$

$S_1 = \frac{1}{4}$

$S_2 = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

$S_3 = \frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} = \frac{7}{4}$

$S_4 = \frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} + \frac{2^3}{4} = \frac{15}{4}$

$$S_n = \frac{a - ar^n}{(1-r)} = \frac{\left(\frac{1}{4}\right) - \left(\frac{1}{4}\right)2^n}{1-2} = \boxed{-\frac{1}{4} + \frac{1}{4}(2^n)}$$

$\lim_{n\to\infty} S_n = \infty$, diverges

④ $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{k+2}$ geometric $\quad a = \left(\frac{2}{3}\right)^3 \quad r = \frac{2}{3}$

$$\text{sum} = \frac{\left(\frac{2}{3}\right)^3}{1-\frac{2}{3}} = \boxed{\frac{8}{9}}$$ converges

⑥ $\sum_{k=1}^{\infty} \left(-\frac{3}{2}\right)^{k+1} \quad r = -\frac{3}{2}$

$|r| > 1$ $\boxed{\therefore \text{diverges}}$

⑧ $$\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right) = \frac{1}{2} - \underbrace{\frac{1}{2^{n+1}}}_{\to 0 \text{ as } n \text{ increases}} \Rightarrow \lim_{n\to\infty} S_n = \frac{1}{2}$$

telescoping series

$\boxed{\text{converges}}$

Section 11.3

(17) $5.373737\cdots = 5 + 0.37 + 0.0037 + 0.000037 + \cdots$

$$= 5 + \frac{0.37}{1-0.01} = 5 + \frac{37}{99} = \boxed{\frac{532}{99}}$$

(20) $0.451141414\ldots = 0.451 + 0.00014 + 0.0000014 + \cdots$

$$= 0.451 + \frac{0.00014}{1-0.01} = \boxed{\frac{44663}{99000}}$$

(24) volume $= 1^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{4}\right)^3 + \cdots + \left(\frac{1}{2^n}\right)^3 + \cdots$

$$= 1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \cdots + \left(\frac{1}{8}\right)^n + \cdots$$

$$= \frac{1}{1-\frac{1}{8}} = \boxed{\frac{8}{7}}$$

(35) The series converges to $1/(1-x)$ only if $-1<x<1$

Section 11.4

Theorem 11.4.3

(2) b) $\displaystyle\sum_{k=1}^{\infty}\left[7^{-k}3^{k+1} - \frac{2^{k+1}}{5^k}\right] = \frac{9}{4} - \frac{4}{3} = \boxed{\frac{11}{12}}$

with $a = \frac{9}{7}$, $r = \frac{3}{7}$, geometric, $\displaystyle\sum_{k=1}^{\infty} 7^{-k}3^{k+1} = \frac{\frac{9}{7}}{1-\frac{3}{7}} = \frac{9}{4}$

with $a = \frac{4}{5}$, $r = \frac{2}{5}$, geometric, $\displaystyle\sum_{k=1}^{\infty} \frac{2^{k+1}}{5^k} = \frac{\frac{4}{5}}{1-\frac{2}{5}} = \frac{4}{3}$

Section 11.4

(4) a) $\sum_{k=1}^{\infty} k^{-4/3}$ $p = \frac{4}{3}$, converges

b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k}}$ $p = \frac{1}{4}$, diverges

c) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^5}}$ $p = \frac{5}{3}$, converges

d) $\sum_{k=1}^{\infty} \frac{1}{k^{\pi}}$ $p = \pi$, converges

(6) a) $\sum_{k=1}^{\infty} \frac{k}{e^k}$, $\lim_{k \to \infty} \frac{k}{e^k} = 0$; no information

b) $\sum_{k=1}^{\infty} \ln k$, $\lim_{k \to \infty} \ln k = \infty$; diverges

c) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$, $\lim_{k \to \infty} \frac{1}{\sqrt{k}} = 0$; no information

d) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k}+3}$, $\lim_{k \to \infty} \frac{\sqrt{k}}{\sqrt{k}+3} = 1$; diverges

(9) $\sum_{k=1}^{\infty} \frac{1}{k+6} = \sum_{k=7}^{\infty} \frac{1}{k}$, diverges because harmonic series diverges

Section 11.4

(10) $\displaystyle\sum_{k=1}^{\infty} \frac{3}{5k} = \frac{3}{5} \sum_{k=1}^{\infty} \frac{1}{k}$, diverges because harmonic series $\boxed{\text{diverges}}$

(15) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\ln(k+1)}$ $\quad \displaystyle\underbrace{\lim_{k\to\infty} \frac{k}{\ln(k+1)} = \lim_{k\to\infty} \frac{1}{1/(k+1)}}_{\text{L' Hôpital's Rule}} = \infty$

$\boxed{\therefore \text{diverges}}$

because $\displaystyle\lim_{k\to\infty} \neq 0$

(23) $\displaystyle\sum_{k=5}^{\infty} 7k^{-1.01}$, p-series with $p>1$, $\boxed{\text{converges}}$