Solution Set week # 2

11.6 2) a) $\sum_{k=2}^{\infty} \frac{k+1}{k^2-k}$ compare $\frac{k+1}{k^2-k}$ to $\frac{k}{k^2}$ $\frac{k+1}{k^2-k} > \frac{k}{k^2} = \frac{1}{k}$

$\sum_{k=2}^{\infty} \frac{1}{k}$ diverges therefore $\sum_{k=2}^{\infty} \frac{k+1}{k^2-k}$ also <u>diverges</u>.

b) $\sum_{k=1}^{\infty} \frac{2}{k^4+k}$ compare to $\frac{2}{k^4}$ (which converges due to p-series test)

$\frac{2}{k^4+k} < \frac{2}{k^4}$ $\therefore$ $\frac{2}{k^4+k}$ also <u>converges</u>

7) $\sum_{k=1}^{\infty} \frac{5}{3^k+1}$ compare to $\sum_{k=1}^{\infty} \frac{5}{3^k}$ $\rho = \lim_{k\to\infty} \frac{3^k}{3^k+1} = 1$ Knowing that $\sum_{k=1}^{\infty} \frac{5}{3^k}$ converges, $\sum_{k=1}^{\infty} \frac{5}{3^k+1}$ also does <u>converge</u>.

14) <u>Ratio Test</u>

$\sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k$ $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k} \Rightarrow \rho = \lim_{k\to\infty} \frac{(k+1)\left(\frac{1}{2}\right)^{k+1}}{k\left(\frac{1}{2}\right)^k} = \frac{1}{2} \lim_{k\to\infty} \frac{k+1}{k} = \frac{1}{2} < 1$ $\boxed{\text{converges}}$

24) $\sum_{k=1}^{\infty} \frac{k!\,10^k}{3^k}$ $\boxed{\text{Ratio Test}}$ $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k} \Rightarrow \rho = \lim_{k\to\infty} \frac{(k+1)!\,10^{k+1}\,(3^k)}{3^{k+1}\,k!\,10^k} = \lim_{k\to\infty} \frac{10(k+1)}{3} = \infty$ $\boxed{\text{Series Diverges}}$

29) $\boxed{\text{Limit Comparison Test}}$ $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$ with $\sum_{k=1}^{\infty} \frac{1}{k}$ (divergent series). $\rho = \lim_{k\to\infty} \frac{k}{\sqrt{k^2+k}} = 1$ $\therefore$ Since $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$ $\boxed{\text{also diverges}}$

see solution of Example 3(c) on the top of p. 660.

34) $\sum_{k=1}^{\infty} \frac{k!}{k^k} \xRightarrow{\boxed{\text{Ratio Test}}}$ $\rho = \lim_{k\to\infty} \frac{(k+1)!}{(k+1)^{k+1}} \frac{k^k}{k!} = \overbrace{\lim_{k\to\infty} \frac{k^k}{(k+1)^k} = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^{-k}} = \frac{1}{e} < 1$ so it $\boxed{\text{converges}}$

44) $\sum_{k=1}^{\infty} \frac{(k!)^2\, 2^k}{(2k+2)!} \xRightarrow{\text{Ratio test}}$ so $\rho = \lim_{k\to\infty} \frac{((k+1)!)^2\, 2^{k+1}}{(2k+4)!} \frac{(2k+2)!}{(k!)^2\, 2^k} = \lim_{k\to\infty} \frac{2(k+1)^2}{(2k+3)(2k+4)} = \boxed{\frac{1}{2} \text{ Series converges}}$

$= \lim_{k\to\infty} \frac{2k^2+4k+2}{4k^2+14k+12}$

11.7 3) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$ $\boxed{\text{diverges because } \lim_{k\to\infty} \frac{k+1}{3k+1} = \frac{1}{3} \neq 0}$

6) $\sum_{k=3}^{\infty} (-1)^k \frac{\ln k}{k}$ $\boxed{\text{converges}}$ $\lim_{k\to\infty} \frac{\ln k}{k} = 0$ and $\left\{\frac{\ln k}{k}\right\}$ is decreasing.

16) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$ is $\boxed{\text{absolutely convergent}}$ because $\sum_{k=1}^{\infty} \frac{1}{k!}$ is convergent $\lim_{k\to\infty} \frac{k!}{(k+1)!} = \lim_{k\to\infty} \frac{1}{k+1} = 0$

or 8) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k}{k!}$ $\boxed{\text{converges}}$ by ratio test: $\lim_{k\to\infty} \left| \frac{(-1)^{k+2}\, 2^{k+1}}{(k+1)!} \Big/ \frac{(-1)^{k+1}\, 2^k}{k!} \right| = \lim_{k\to\infty} \left(\frac{2}{k+1}\right) = 0 < 1$

11.8 1) $\sum_{k=0}^{\infty} (-1)^k x^k$ $\rho = \lim_{k\to\infty} \left|\frac{x^{k+1}}{x^k}\right| = \lim_{k\to\infty} |x|$ so the interval of convergence is $-1 < x < 1$

It is a geometric series with $a = 1$ $r = -x$ so it converges to $\frac{1}{1+x}$ and diverges for $x = \pm 1$ (simply check by plugging in $x = \pm 1$)

2) $\sum_{k=0}^{\infty} x^{2k}$ $\rho = \lim_{k\to\infty} \left|\frac{x^{2k+2}}{x^{2k}}\right| = \lim_{k\to\infty} |x^2|$ so the interval of convergence is $-1 < x < 1$

It does not converge for $x = \pm 1$ and for $-1 < x < 1$, the series converges to $\frac{1}{1-x^2}$ $\left(\begin{array}{l}\text{geometric series}\\ a = 1\\ r = x^2\end{array}\right)$

5) a) geometric series $\sum_{k=0}^{\infty} \frac{x^k}{2^k}(-1)^k$ $\rho = \lim_{k\to\infty} \left|\frac{x^{k+1}}{2^{k+1}} \frac{2^k}{x^k}\right| = \lim_{k\to\infty} \left|\frac{x}{2}\right|$ so the interval of convergence is $-2 < x < 2$

$a = 1$
$r = -\frac{x}{2}$ $\frac{1}{1+\frac{x}{2}} = \frac{2}{2+x}$. The series diverges for $x = \pm 2$

b) $f(0) = 1$ $f(1) = \frac{2}{3}$ (plugging by into the closed form $\frac{2}{2+x}$).

10) $\sum_{k=0}^{\infty} \frac{k!}{2^k} x^k$ $\rho = \lim_{k\to\infty} \frac{(k+1)!\,(x^{k+1})2^k}{2^{k+1}\;k!\;x^k} = \lim_{k\to\infty} \left|\frac{(k+1)x}{2}\right| = \infty$ Radius of convergence $= 0$
interval of convergence is $x = 0$

16) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$ $\rho = \lim_{k\to\infty} \left|\frac{x^{2k+2}}{(2k+2)!} \frac{2k!}{x^{2k}}\right| = \lim_{k\to\infty} \left|\frac{x^2}{(2k+1)(2k+2)}\right| = 0$ Radius of convergence $= \infty$
interval of convergence is
$-\infty < x < \infty$
or $(-\infty, \infty)$