Math 1b Homework Solution Set Oct. 28 – Nov 3

7.3

2) a) $\int \sec^2(4x+1)\,dx = \frac{1}{4}\int \sec^2 u\,du = \frac{1}{4}(\tan u + C) = \boxed{\frac{1}{4}\tan(4x+1) + C}$

$u = 4x+1$

$du = 4dx$

b) $\int y\sqrt{1+2y^2}\,dy = \frac{1}{4}\int(\sqrt{1+2y^2})\,4y\,dy = \frac{1}{4}\int\sqrt{u}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$

$u = 1+2y^2$

$= \boxed{\frac{1}{6}(1+2y^2)^{3/2} + C}$

$du = 4y\,dy$

c) $\int\sqrt{\sin\pi\theta}\,\cos\pi\theta\,d\theta = \frac{1}{\pi}\int\sqrt{\sin\pi\theta}\,(\pi\cos\pi\theta)\,d\theta = \frac{1}{\pi}\int\sqrt{u}\,du = \frac{2}{3\pi}u^{3/2} + C$

$u = \sin\pi\theta$

$= \boxed{\frac{2(\sin\pi\theta)^{3/2}}{3\pi} + C}$

$du = (\pi\cos\pi\theta)\,d\theta$

d) $\int(2x+7)(x^2+7x+3)^{4/3}\,dx = \int(x^2+7x+3)^{4/3}(2x+7)\,dx = \int u^{4/3}\,du = \frac{3}{7}u^{7/3} + C$

$u = x^2+7x+3$

$= \boxed{\frac{3}{7}(x^2+7x+3)^{7/3} + C}$

$du = (2x+7)\,dx$

e) $\int\frac{e^x}{1+e^x}\,dx = \int\frac{e^x\,dx}{1+e^x} = \int\frac{du}{u} = \ln u + C = \boxed{\ln(1+e^x) + C}$

$u = 1+e^x$

$du = e^x\,dx$

6) $\int\frac{dx}{2x} = \frac{1}{2}\int\frac{2dx}{2x} = \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\ln u + C = \boxed{\frac{1}{2}\ln 2x + C}$

$u = 2x$

$du = 2dx$

8) $\int(3x-1)^5\,dx = \frac{1}{3}\int(3x-1)^5\,3dx = \frac{1}{3}\int u^5\,du = \frac{1}{18}u^6 + C = \boxed{\frac{1}{18}(3x-1)^6 + C}$

$u = 3x-1$

$du = 3dx$

14) $\int\frac{x}{\sqrt{4-5x^2}}\,dx = \frac{1}{-5}\int\frac{-5x}{\sqrt{4-5x^2}}\,dx = \frac{-1}{5}\int du = \frac{-u}{5} + C = \boxed{\frac{\sqrt{4-5x^2}}{-5} + C}$

$u = \sqrt{4-5x^2} \Rightarrow du = \frac{-5x}{\sqrt{4-5x^2}}\,dx$

23) $\int \frac{\sin(5/x)}{x^2}\,dx = \frac{1}{-5}\int \sin(5/x)\left(\frac{-5}{x^2}\right)dx = \frac{-1}{5}\int \sin u\,du = \frac{1}{5}\cos u + C$

$u = 5/x$

$du = \frac{-5}{x^2}$

$= \boxed{\frac{1}{5}\cos\left(\frac{5}{x}\right) + C}$

50) a) method 1:

$\int (5x-1)^2 dx = \int (25x^2 + -10x + 1)\,dx = \frac{25}{3}x^3 - 5x^2 + x + C$

method 2:

$\int (5x-1)^2 dx = \frac{1}{5}\int (5x-1)^2\, 5\,dx = \frac{1}{5}\int u^2 du \;\; \frac{1}{15}(5x-1)^3 + C'$

$u = 5x-1$

$du = 5\,dx$

$\frac{1}{5}\int u^2 du = \frac{1}{15}u^3 + C' =$

$\Rightarrow \int (5x-1)^2 dx = \frac{1}{15}(125x^3 - 75x^2 + 15x - 1) + C'$

$= \frac{25}{3}x^3 - 5x^2 + x + (C' - 1)$

b) the two answers are seen to be equivalent if we let $C' - 1 = C$, which is fine since C' & C are both arbitrary constants.

7.8

~~29)~~ $\cancel{\int_{-3\pi/4}^{\pi/4} x \sin 2x\, dx}$

29) $\int_{-3\pi/4}^{\pi/4} \sin x \cos x\, dx = \int_{u=\sin -3\pi/4}^{u=\sin \pi/4} u\,du = \int_{-\sqrt{2}/2}^{\sqrt{2}/2} u\,du = \left[\frac{1}{2}u^2\right]_{-\sqrt{2}/2}^{\sqrt{2}/2} = \boxed{0}$

$u = \sin x$

$du = \cos x\, dx$

38) $\int_1^{\sqrt{2}} x e^{-x^2} dx = \frac{1}{-2}\int_1^{\sqrt{2}} -2x e^{-2x} dx = -\frac{1}{2}\int_{u=\frac{1}{e}}^{u=\frac{1}{e^2}} du = \frac{-1}{2}[u]_{\frac{1}{e}}^{\frac{1}{e^2}} = \boxed{\frac{1}{2}\left(\frac{1}{e} - \frac{1}{e^2}\right)}$

$u = e^{-x^2}$

$du = -2x e^{-x^2} dx$

9.2

5) $\int x \sin 2x \, dx = uv - \int v \, du = -\frac{x}{2}\cos 2x - \int -\frac{1}{2}\cos 2x \, dx = \boxed{-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x + C}$

$u = x \qquad dv = \sin 2x \, dx$

$du = dx \qquad v = -\frac{1}{2}\cos 2x$

8) $\int x^2 \sin x \, dx = -x^2\cos x - \int -2x\cos x \, dx = -x^2 \cos x + 2\int x\cos x \, dx$

$u = x^2 \qquad dv = \sin x \, dx$

$du = 2x\,dx \qquad v = -\cos x$

side note: Int. by parts again

$\int x\cos x \, dx = +x\sin x + \int -\sin x \, dx = +x\sin x + +\cos x$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = +\sin x$

$\Rightarrow \int x^2 \sin x \, dx = \boxed{-x^2\cos x + 2x\sin x + 2\cos x}$

11) $\int (\ln x)^2 dx = x(\ln x)^2 - \int 2\ln x \, dx = x(\ln x)^2 - 2\int \ln x \, dx = \boxed{x(\ln x)^2 - 2(x\ln x - x) + C}$

$u = (\ln x)^2 \qquad dV = dx$

$du = \frac{2}{x}\ln x \, dx \qquad v = x$

note $\int \ln x \, dx$ is solved on page 518 ex#3

$\cancel{20)\ \int e^{2x}\cos 3x \, dx = \frac{e^{2x}}{3}\sin 3x - \int \frac{2}{3}e^{2x}\sin 3x \, dx = \frac{e^{2x}}{3}\sin 3x - \frac{2}{3}\left(\frac{-e^{2x}}{3}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x \, dx\right)}$

$\cancel{u = e^{2x} \qquad dv = \cos 3x \, dx}$ ~~Int. by parts again~~ $\cancel{u = e^{2x} \qquad dv = \sin 3x \, dx}$

$\cancel{du = 2e^{2x}dx \qquad v = \frac{1}{3}\sin 3x}$ $\cancel{du = 2e^{2x} \qquad v = -\frac{1}{3}\cos 3x}$

$\cancel{\Rightarrow \int e^{2x}}$

20) ① $\int e^{2x}\cos 3x \, dx = \frac{e^{2x}}{3}\sin 3x - \int \frac{2}{3}e^{2x}\sin 3x \, dx$

$u = e^{2x} \qquad dv = \cos 3x \, dx$

$du = 2e^{2x}dx \qquad v = \frac{1}{3}\sin 3x$

Int. by parts again to get

② $\int \frac{2}{3}e^{2x}\sin 3x \, dx = \frac{2}{3}\left(\frac{-e^{2x}}{3}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x \, dx\right)$

combine ① & ② to get

$\int e^{2x}\cos 3x \, dx = \frac{e^{2x}}{3}\sin 3x + \frac{2e^{2x}}{9}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x \, dx$

continued on next page

9.2 #20 cont.

$$\Rightarrow \left(1+\frac{4}{9}\right)\int e^{x}\cos 3x\,dx = \frac{e^{2x}}{3}\sin 3x + \frac{2e^{2x}}{9}\cos 3x$$

$$\Rightarrow \boxed{\int e^{2x}\cos 3x\,dx = \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + C}$$

30) $$\int_0^2 xe^{2x}dx = \frac{x}{2}e^{2x}\Big|_0^2 - \frac{1}{2}\int_0^2 e^{2x}dx = e^4 - \frac{1}{4}e^{2x}\Big|_0^2 = e^4 - \frac{e^4}{4} + \frac{1}{4}$$

$u = x \qquad dv = e^{2x}dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$$\boxed{= \frac{3}{4}e^4 + \frac{1}{4}}$$

41) a) $$\int e^{\sqrt{x}}dx = \int e^{u}2u\,du = 2ue^{u} - 2e^{u} + C = \boxed{2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C}$$

$u = \sqrt{x} \qquad \Rightarrow dx = 2\sqrt{x}\,du$

$du = \frac{1}{2\sqrt{x}}dx \qquad dx = 2u\,du$

Int. by parts: $w = 2u \qquad dv = e^{u}$

$dw = 2du \qquad v = e^{u}$

b) $$\int \cos\sqrt{x}\,dx = \int \cos u\,2u\,du = 2u\sin u - 2\int \sin u\,du = 2u\sin u + 2\cos u + C$$

$u = \sqrt{x} \qquad \Rightarrow dx = 2\sqrt{x}\,du$

$du = \frac{1}{2\sqrt{x}}dx \qquad dx = 2u\,du$

int. by parts: $w = 2u \qquad dv = \cos u\,du$

$dw = 2du \qquad v = \sin u$

$$\Rightarrow \boxed{\int \cos\sqrt{x}\,dx = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C}$$

9.5

14) $$\int \frac{dx}{x(x^2-1)} = \int\left(\frac{a}{x} + \frac{b}{x-1} + \frac{c}{x+1}\right)dx = \int \frac{ax^2 - a + bx^2 + bx + cx^2 - cx}{x(x^2-1)}dx$$

partial fractions

note a, b, c arbitrary constants

$$\Rightarrow ax^2 + bx^2 + cx^2 + bx - cx - a = 1 \quad \Rightarrow \quad a = -1,\; b = \frac{1}{2},\; c = \frac{1}{2}$$

$$\Rightarrow \int \frac{dx}{x(x^2-1)} = \int\left(\frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}\right)dx = \frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{1+x}dx - \int \frac{1}{x}dx$$

$$= \frac{1}{2}\ln(x-1) + \frac{1}{2}\ln(x+1) - \ln x + C$$

$$\boxed{= \ln\frac{\sqrt{x^2-1}}{x} + C}$$

16) $$\int \frac{x^2-4}{x-1}\,dx = \int\left(x+1+\frac{-3}{x-1}\right)dx = \int x\,dx + \int dx - 3\int\frac{1}{x-1}\,dx = \boxed{\tfrac{1}{2}x^2 + x - 3\ln(x-1) + C}$$

note:
$$\begin{array}{r|l} & x+1 \\ x-1 & x^2+0x-4 \\ & -\;x^2-x \\ \hline & x-4 \\ & -\;x-1 \\ \hline & -3 \end{array}$$

18) $$\int\frac{x^2}{x^2-3x+2}\,dx = \int\left(1+\frac{3x-2}{x^2-3x+2}\right)dx = \int\left(1+\frac{3x-2}{(x-2)(x-1)}\right)dx = \int\left(1+\frac{4}{x-2}+\frac{-1}{x-1}\right)dx$$

note:
$$\begin{array}{r|l} & 1 \\ x^2-3x+2 & x^2+0 \\ & -\;x^2-3x+2 \\ \hline & 3x-2 \end{array}$$

note:
$$\frac{a}{x-2}+\frac{b}{x-1} = \frac{(a+b)x-(2b+a)}{(x-2)(x-1)} = \frac{3x-2}{(x-2)(x-1)}$$

$\Rightarrow a = 4$

$b = -1$

$$\Rightarrow \boxed{\int\frac{x^2}{x^2-3x+2}\,dx = x + 4\ln(x-2) - \ln(x-1) + C}$$

33) $$\int\frac{\cos\theta\,d\theta}{\sin^2\theta+4\sin\theta-5} = \int\frac{du}{u^2+4u-5} = \int\frac{du}{(u+5)(u-1)} = \int\left(\frac{-1}{6(u+5)}+\frac{1}{6(u-1)}\right)du$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

note
$$\frac{a}{u+5}+\frac{b}{u-1} = \frac{(a+b)u+(5b-a)}{(u+5)(u-1)} = \frac{1}{(u+5)(u-1)}$$

$\Rightarrow a = -\frac{1}{6}$

$b = \frac{1}{6}$

$$\Rightarrow \int\left(\frac{1}{6(u-1)}-\frac{1}{6(u+5)}\right)dx = \frac{1}{6}\left(\ln(u-1)-\ln(u+5)\right) = \frac{1}{6}\ln\frac{u-1}{u+5} + C$$

$$\Rightarrow \boxed{\int\frac{\cos\theta\,d\theta}{\sin^2\theta+4\sin\theta-5} = \frac{1}{6}\ln\frac{\sin\theta-1}{\sin\theta+5} + C}$$

don't forget the constant (I almost did)