Math 16 Homework Solutions

Nov 12 – Nov 18

8.5

2) $y = x^{1/2}$, $1 \le x \le 4$ about x-axis

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$$

$$S = \int_1^4 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^4 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx$$

$$= \int_1^4 2\pi\sqrt{x+\frac{1}{4}}\,dx = 2\pi\int_1^4 (x+1/4)^{1/2}\,dx$$

$$= 2\pi\cdot\frac{2}{3}\cdot\left[(x+1/4)^{3/2}\right]_1^4 = \boxed{\frac{4\pi}{3}\left(\left(\frac{17}{4}\right)^{3/2}-\left(\frac{5}{4}\right)^{3/2}\right)}$$

4) $x = \sqrt[3]{y}$, $1 \le y \le 8$ about x-axis

$y = x^3$ $\quad 1 \le x \le 2$

$$\frac{dy}{dx} = 3x^2 \Rightarrow \left(\frac{dy}{dx}\right)^2 = 9x^4$$

$$S = \int_1^2 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^2 2\pi x^3\sqrt{1+9x^4}\,dx$$

$u = 1+9x^4 \Rightarrow du = 36x^3\,dx$

$$\int \frac{2\pi x^3}{36x^3}u^{1/2}\,du = \frac{\pi}{18}\int u^{1/2}\,du = \frac{\pi}{18}\cdot\frac{2}{3}\cdot u^{3/2}$$

$$\frac{\pi}{27}\cdot(1+9x^4)^{3/2}\Big|_1^2 = \boxed{\frac{\pi}{27}\left(145^{3/2}-10^{3/2}\right)}$$

8.5

8) $x = 2\sqrt{1-y}$ $\quad -1 \le y \le 0$ about y-axis

$$\frac{dx}{dy} = -(1-y)^{-1/2} \Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{1}{1-y}$$

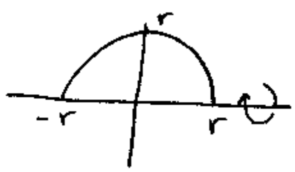
$$S = \int_{-1}^{0} 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dx = \int_{-1}^{0} 2\pi 2\sqrt{1-y}\sqrt{1+\frac{1}{1-y}}\, dy$$

$$= \int_{-1}^{0} 4\pi\sqrt{1-y+1}\, dy = 4\pi\int_{-1}^{0}(2-y)^{1/2}\, dy$$

$$= -\frac{4\pi \cdot 2}{3}(2-y)^{3/2}\Big|_{-1}^{0} = \frac{-8\pi}{3}\left(2^{3/2} - (3)^{3/2}\right)$$

$$= \boxed{\frac{8\pi}{3}\left(3^{3/2} - 2^{3/2}\right)}$$

18)



$y = \sqrt{r^2 - x^2}$ $\quad -r \le x \le r$

Rotate about the x-axis

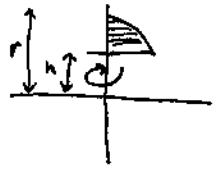
$$\frac{dy}{dx} = -x(r^2-x^2)^{-1/2} \Rightarrow \left(\frac{dy}{dx}\right)^2 = x^2/(r^2-x^2)$$

$$S = \int_{-r}^{r} 2\pi\sqrt{r^2-x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\, dx = \int_{-r}^{r} 2\pi\sqrt{r^2-x^2+x^2}\, dx$$

$$= \int_{-r}^{r} 2\pi\sqrt{r^2}\, dx = \int_{-r}^{r} 2\pi r\, dx = 2\pi r\, x\Big|_{-r}^{r}$$

$$= 2\pi r\,(r-(-r)) = 2\pi r(2r) = \boxed{4\pi r^2}$$

8.5

19) a) 

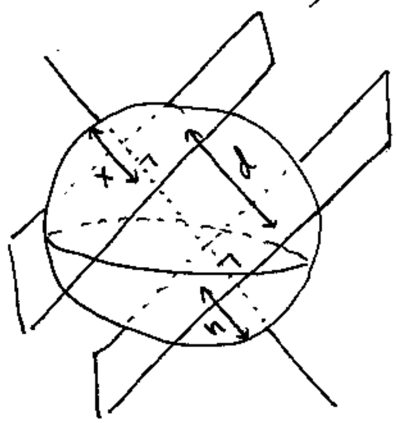
$x = \sqrt{r^2 - y^2}$ $\quad r-h \le y \le r$ about y-axis

$\frac{dx}{dy} = -y(r^2-y^2)^{-1/2} \Rightarrow \left(\frac{dx}{dy}\right)^2 = y^2/(r^2-y^2)$

$$S = \int_{r-h}^{r} 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy = \int_{r-h}^{r} 2\pi \sqrt{r^2-y^2}\sqrt{1+\frac{y^2}{r^2-y^2}}\, dy$$

$$= \int_{r-h}^{r} 2\pi \sqrt{r^2-y^2+y^2}\, dy = \int_{r-h}^{r} 2\pi r\, dy = 2\pi r \; y\Big|_{r-h}^{r}$$

$$= 2\pi r(r-r+h) = \boxed{2\pi r h}$$

b) 

Surface of the zone
= Surface of the sphere − the surfaces of the 2 "caps".

$S = 4\pi r^2 - 2\pi r x - 2\pi r h$

But, $h = 2r - d - x$

$$\therefore \quad S = 4\pi r^2 - 2\pi r x - 2\pi r(2r-d-x)$$

$$= 4\pi r^2 - 2\pi r x - 4\pi r^2 + 2\pi r d + 2\pi r x$$

$$= \boxed{2\pi r d}$$

8.6

2) $W = \int_0^5 F(x)\,dx = \int_0^2 40\,dx + \int_2^5 40 - \frac{40}{3}(x-2)\,dx$

$= 40x\Big|_0^2 + 40x\Big|_2^5 - \frac{40}{2\cdot 3}(x-2)^2\Big|_2^5$

$= 80 + 120 - \frac{40}{6}(9-0) = 80+120-60 = \boxed{140\ Nm}$

4) a) $F(x) = kx$, $F(0.05) = 0.05k = 45$

distance stretched $= 5cm = .05$ meters → so $k = 900\ N/m$

b) $W = \int_0^{0.03} 900x\,dx = 450x^2\Big|_0^{.03} = 0.405$ Joules

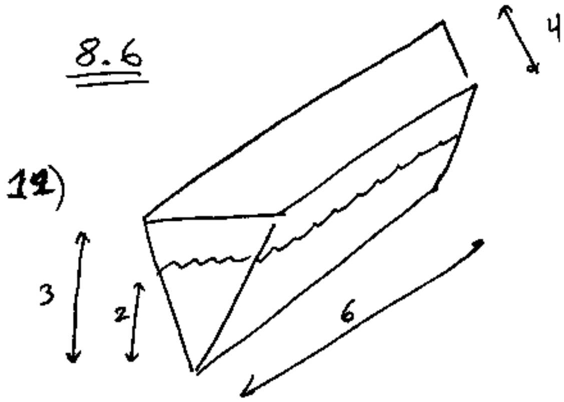
c) $W = \int_{0.05}^{0.10} 900x\,dx = 450x^2\Big|_{.05}^{.10} = 3.375$ Joules

7) $W = \int_0^1 kx\,dx = 10 \Rightarrow \frac{kx^2}{2}\Big|_0^1 = 10 \Rightarrow \frac{k}{2} = 10$

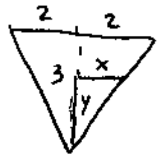
$\Rightarrow \boxed{k = \frac{20\ lb}{ft}}$

8.6

13)



$$dW = dV \cdot 9810 \cdot (3-y)$$
$$= dA \cdot dy \cdot 9810\,(3-y)$$
$$= 2x \cdot 6 \cdot dy \cdot 9810\,(3-y)$$



$$\frac{2}{3} = \frac{x}{y} \Rightarrow x = \frac{2y}{3}$$

$$\therefore \; dW = \frac{24y}{3}(3-y) \cdot 9810 \; dy$$

$$W = \int dW = \int_0^2 9810 \cdot \left(24y - \frac{24y^2}{3}\right) dy = 9810 \cdot \left(12y^2 - \frac{24}{9}y^3\right)\Big|_0^2$$

$$= 9810 \cdot \frac{80}{3} = \boxed{261{,}600 \text{ J}}$$

14) $$W = \int_0^{3{,}000} 43 - \frac{2}{1{,}000}h \; dh$$

$$= 43h\Big|_0^{3{,}000} - \frac{h^2}{1{,}000}\Big|_0^{3{,}000}$$

$$= 129{,}000 - 9{,}000 = \boxed{120{,}000 \text{ tons} \cdot \text{ft}}$$

15) $$W = \int_0^{100} \underbrace{(100-y)}_{\text{length}} \cdot \underbrace{15}_{\text{weight/length}} \, dy = \int_0^{100} 1{,}500 - 15y \; dy$$

$$= 1{,}500y\Big|_0^{100} - \frac{15}{2}y^2\Big|_0^{100} = 150{,}000 - 75{,}000$$

$$= \boxed{75{,}000 \text{ lb} \cdot \text{ft}}$$

8.6

17) $w(x) = K/x^2$

a) 150 lb on surface $\Rightarrow 150 = \dfrac{K}{4{,}000^2} \Rightarrow K = 2{,}400{,}000{,}000$

$$\Rightarrow \boxed{w(x) = \frac{2{,}400{,}000{,}000}{x^2}}$$

b) $6{,}000 = \dfrac{K}{4{,}000^2} \Rightarrow K = 9.6 \times 10^{10}$

$$\boxed{w(x) = \frac{9.6 \times 10^{10}}{(4{,}000 + x)^2}}$$

c) $$W = \int_{4{,}000}^{5{,}000} \frac{9.6 \times 10^{10}}{x^2}\,dx = -9.6 \times 10^{10} \cdot \frac{1}{x}\Big|_{4{,}000}^{5{,}000}$$

$$= -9.6 \times 10^{10}\left(\frac{1}{5{,}000} - \frac{1}{4{,}000}\right) = \boxed{4{,}800{,}000 \text{ lb}\cdot\text{miles}}$$

$= 2.5344 \times 10^{10}$ ft.-lb.s

18) a) $20 = \dfrac{K}{1080^2} \Rightarrow K = 23{,}328{,}000$

$$\boxed{w(x) = \frac{23{,}328{,}000}{(1080 + x)^2}}$$

b) $$W = \int_0^{10.8} \frac{23{,}328{,}000}{(1080 + x)^2}\,dx = -23{,}328{,}000 \left.\frac{1}{1080 + x}\right|_0^{10.8}$$

$$= -23{,}328{,}000\left(\frac{1}{1090.8} - \frac{1}{1080}\right) = \boxed{\frac{21{,}600}{101} \text{ lb}\cdot\text{miles}}$$

$= 213.86$ mi$\cdot$lb

$= 1{,}129{,}188$ ft. lbs

8.7

For problems 4-8, let $x=0$ at the surface

4) $F=\int_1^3 \rho\, h(x)\, w(x)\, dx \qquad h(x)=x \quad w(x)=4 \quad \rho=9810$

$F=\int_1^3 9810\cdot 4\cdot x\, dx = 39{,}240\, \frac{x^2}{2}\Big|_1^3 = \frac{39{,}240}{2}(9-1)$

$= 39{,}240\cdot 4 = \boxed{156{,}960\ N}$

5) $F=\int_0^5 \rho\, h(x)\, w(x)\, dx \qquad h(x)=x \quad w(x)=2\sqrt{25-x^2} \quad \rho=9810$

$=\int_0^5 9810\cdot 2\cdot x\cdot\sqrt{25-x^2}\, dx \qquad u=25-x^2$

$= -9810\int u^{1/2} = \frac{-9810\cdot 2}{3}\left(25-x^2\right)^{3/2}\Big|_0^5$

$= 6540\left(25^{3/2}-0^{3/2}\right) = \boxed{817{,}500\ N}$

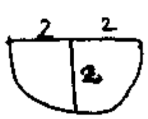
8) $F=\int_4^{12} \rho\, h(x)\, w(x)\, dx \qquad h(x)=x \quad w(x)=24-(x-4) \quad \rho=62.4$

$=\int_4^{12} 62.4\ x\ (24-x+4)\, dx = 62.4\int_4^{12} x(28-x)\, dx$

$= 62.4\int_4^{12} 28x-x^2\, dx = 62.4\left[14x^2-\frac{x^3}{3}\right]\Big|_4^{12} = \frac{62.4\times 3{,}712}{3}$

$= \boxed{77{,}209.6\ lb}$

10)



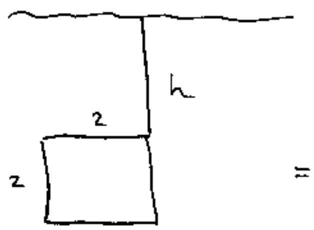
$F=\int_0^2 \rho\ h(x)\ w(x)\ dx \qquad \rho=50 \quad h(x)=x \quad w(x)=2\sqrt{4-x^2}$

$=\int_0^2 100\ x\sqrt{4-x^2}\, dx \qquad u=4-x^2,\ du=-2x\,dx \qquad = -50\int u^{1/2}\, du$

$= \frac{-50\cdot 2}{3}\left(4-x^2\right)^{3/2}\Big|_0^2 = \frac{100}{3}\ 4^{3/2} = \boxed{\frac{800}{3}\ lb}$

8.7

14)



$$F = \int_h^{h+2} \rho_0 \, h(x)\, w(x) \, dx \qquad h(x) = x, \quad w(x) = 2$$

$$= \int_h^{h+2} \rho_0 \, 2x \, dx = \rho_0 \, x^2 \Big|_h^{h+2}$$

$$= \rho_0 \left((h+2)^2 - h^2 \right) = \boxed{\rho_0 \left(4h + 4 \right)}$$