Exam #1 Review Worksheet Part I

Math 1B
October '99

Topics covered: - Series (1) Definition of Convergence
- Examples (2) Geometric Series
(3) P-Series
(4) Alternating Series
- Tests for Convergence
(5) - Divergence Test
(6) - Comparison Tests
(7) - Ratio Test (for absolute convergence)

in Review Part II { - Power Series
- Taylor/Maclaurin Series

(1) **Series**: $a_1 + a_2 + a_3 + \cdots = \sum_{k=1}^{\infty} a_k$

converges if

Ex. 1) Does $3 - 3 + 3 - 3 + \cdots$ converge?

(2) Geometric Series

general form $a + ar + ar^2 + \cdots = \sum_{k=1}^{\infty} ar^k$

- converges to $\frac{a}{1-r}$ when

- diverges when

Example 2) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \cdots = \sum_{k=0}^{\infty} \frac{1}{3 \cdot 2^k}$

Example 3) $\sum_{k=3}^{\infty} 2 \cdot \left(\frac{1}{2}\right)^k$

Review continued

<u>Note!</u> Can always add or take away a <u>finite</u> number of terms from a series without changing whether or not it converges!

<u>Example 4</u> $1+3+5+4+2+1+\frac{1}{2}+\frac{1}{4}+\cdots$

Which of the following converge?

<u>Example 5</u> $\sum_{k=1}^{\infty} \frac{3^{2k}}{2^{3k}}$

<u>Example 6</u> $\sum_{k=1}^{\infty} \frac{2^{3k}}{3^{2k}}$

(3) P-Series

general form:

converges when
diverges for

<u>Example 7</u> $1+\frac{1}{8}+\frac{1}{27}+\cdots = \sum_{k=1}^{\infty} \frac{1}{k^3}$

<u>Example 8</u> for what values of r does $\sum_{k=10}^{\infty} \frac{1}{k^{3r}}$ converge?

(4) Alternating series

general form:

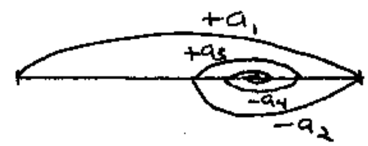
<u>Example 9</u> $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$

<u>Example 10</u> $0.1-0.11+0.111-0.1111+\cdots$

Review continued

Alternating Series Test

Remember the visual mnemonic:



Accuracy of partial sums:

$\Rightarrow$ So if have a converging alternating series $a_1 - a_2 + a_3 - \cdots$ with sum S, then ...

Example 11 Does $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+1}$ converge?

Example 12 Does $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3 + \sqrt{k}}$ converge?

(5) Tests for Convergence – Divergence Test

Example 13 for what values of r does $\sum_{k=1}^{\infty} (k+1) r^2$ converge?

Example 14 does $\sum_{k=1}^{\infty} \frac{k}{3k-5}$ converge?

Example 15 does $\sum_{z=10}^{\infty} \frac{z^2(z+3)}{z^3-1}$ converge?

Review Continued

(6) Comparison Tests

Given two series $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, with a_k, b_k nonnegative and $a_k \le b_k$ for all k, then...

Example 16 Does $\frac{1}{3} + \frac{1}{6} + \frac{1}{11} + \frac{1}{18} + \cdots = \sum_{k=1}^{\infty} \frac{1}{k^2+2}$ converge?

Example 17 Does $\sum_{w=1}^{\infty} \frac{w+1}{w^3}$ converge?

Don't forget you've also got the Limit Comparison Test:

Example 18 $\sum_{t=1}^{\infty} \frac{1}{(t-3.1)^2}$ does this converge?

(7) Ratio Test (for Absolute Convergence)

based on same concept as geometric series $1+r+r^2+\cdots$ converging when $|r|<1$
diverging otherwise

Given a series $\sum_{k=1}^{\infty} a_k \ldots$

Review continued

Example 19 Does $\sum_{k=1}^{\infty} \frac{k+1}{2\cdot k!}$ converge?

Example 20 For what values of u does $\sum_{s=1}^{\infty} \frac{\pi \cdot u^s}{s!}$ converge?

Example 21 Does $\sum_{z=1}^{\infty} \frac{1}{(.8^z)z}$ converge?

Answers to Review Session Examples

Example 1 nope, partial sums bounce between 3 and 0, so limit doesn't exist.

Example 2 $\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\cdots$ first term $=\frac{1}{3}$, ratio $=\frac{1}{2}$ so

$$\text{sum} = \frac{1/3}{1-1/2} = \frac{2}{3}$$

Example 3 first term is $2\cdot\frac{1}{8}=\frac{1}{4}$, ratio $\frac{1}{2}$

($k=3$ starts)

$$\text{sum} = \frac{1/4}{1-1/2} = 2/4 = 1/2$$

Example 4 $= 9+4+2+1+\cdots = 9 +$ geom. series w/ first term 4, ratio $1/2$, $\text{sum}=\frac{4}{1-1/2}=8$

$$\text{sum} = 9+8=17$$

Example 5 geom. series with ratio $9/8 \Rightarrow$ diverges

$$= \sum_{k=1}^{\infty} \frac{9^k}{8^k} = \sum_{k=1}^{\infty}\left(\frac{9}{8}\right)^k$$

Example 6 $= \sum_{k=1}^{\infty}\frac{8^k}{9^k} = \sum_{k=1}^{\infty}\left(\frac{8}{9}\right)^k$ geom. series 1st term $=8/9$, ratio $=8/9$,

$$\text{sum} = \frac{8/9}{1-8/9} = 8$$

Example 7 converges, p-series with $p=3$

Example 8 $\sum_{k=1}^{\infty}\frac{1}{k^{3r}}$ is a p-series with $p=3r$, converges if $p=3r>1$, or when $r>1/3$ (and diverges for $r\le\frac{1}{3}$)

Example 9 Alternating harmonic series $\Rightarrow$ converges

Example 10 does not converge, terms are increasing in absolute value

Example 11 $a_k=\frac{k}{k+1}$, $\lim_{k\to\infty} a_k = 1$, fails alternating series test

diverges also $a_1<a_2<a_3<\cdots$

Example 12 converges $\lim_{k\to\infty}\frac{1}{k^3+\sqrt{k}}=0$, and $a_1=\frac{1}{2}>a_2=\frac{1}{8+\sqrt{2}}>a_3$ etc.

Answers to Review Examples continued

Example 13 only when $r=0$! otherwise $\lim_{k\to\infty}(k+1)r^2 \neq 0$

Example 14 $\lim_{k\to\infty} \frac{k}{3k-5} = \frac{1}{3}$, and so divergence test $\Rightarrow$ diverges

Example 15 $\lim_{z\to\infty} \frac{z^2(z+3)}{z^3-1} = \lim_{z\to\infty} \frac{z^3+3z^2}{z^3-1} = \lim_{z\to\infty} \frac{1+3/z}{1-1/z^3} = 1$, series diverges

Example 16 converges, compare to $\sum_{k=1}^{\infty} \frac{1}{k^2}$ (converges, p-series with $p=2>1$)

Example 17 converges $= \sum_{w=1}^{\infty} \frac{1+1/w}{w^2}$ and $\frac{1+1/w}{w^2} \leq \frac{2}{w^2}$ (converges, p-series, $p=2>1$)

Example 18 compare once again with the p-series $\sum_{t=1}^{\infty} \frac{1}{t^2}$, which converges $\lim_{t\to\infty} \left(\frac{1}{(t-3.1)^2} \Big/ \frac{1}{t^2}\right)$

$= \lim_{t\to\infty} \left(\frac{t^2}{(t-3.1)^2}\right) = 1$, so original series in question also converges

Example 19 $a_k = \frac{k+1}{2\cdot k!}$ $\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{k+1+1}{2\cdot(k+1)!} \Big/ \frac{(k+1)}{2k!}\right|$

$= \lim_{k\to\infty} \left(\left(\frac{k+2}{k+1}\right)\cdot\frac{1}{k+1}\right) = 0 \Rightarrow$ converges

Example 20 $\lim_{s\to\infty} \left|\frac{\pi\cdot u^{s+1}}{(s+1)!} \Big/ \frac{\pi\cdot u^s}{s!}\right| = \lim_{s\to\infty} \left|\frac{u}{s+1}\right| = 0$ for all u, so converges for all u

Example 21 $\lim_{z\to\infty} \left|\frac{1}{.8^{z+1}\cdot(z+1)} \Big/ \frac{1}{.8^z\cdot z}\right| = \lim_{z\to\infty} \left(\frac{1}{.8}\left(\frac{z}{z+1}\right)\right) = \frac{1}{.8} = 1.25 > 1$ diverges