Things to know

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Must know:

$$1 + r + r^2 + r^3 + \cdots = \sum_{k=0}^{\infty} r^k = \frac{1}{1-r} \quad \text{if } |r| < 1$$

Taylor series for $f(x)$ near c.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(c)\,(x-c)^n}{n!}$$ where $f^n(c)$ is the n^{th} derivative of $f(x)$ evaluated at $x = c$

Common series:

	Interval of Con.
$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$	$(-\infty, \infty)$
$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$	$(-\infty, \infty)$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$	$(-\infty, \infty)$
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{k=0}^{\infty} x^k$	$(-1, 1)$

Tests

1) Divergence Test: If terms don't go to zero, the series diverges (if they do, we don't know anything)

2) Ratio Test: $\rho = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right|$ $\rho < 1$ converges; $\rho > 1$ diverges; $\rho = 1$ don't know

3) Alternating Series Test: if: (1) terms alternate, (2) terms $\to 0$, (3) terms abs. val. decreases } then converges

4) Comparison Test: term by term:
- if series < convergent series, then con.
- if series > divergent series, then div.