Math 1ab Summer 2001 - Solutions to Problem Set 1

Problem 1 (2.1 # 2)

We compute the slope of secant line between the indicated points.

(a)

$$\text{slope} = \frac{2948 - 2530}{42 - 36} = \frac{209}{3} \approx 69.7$$

(b)

$$\text{slope} = \frac{2948 - 2661}{42 - 38} = \frac{287}{4} = 71.75$$

(c)

$$\text{slope} = \frac{2948 - 2806}{42 - 40} = 71$$

(d)

$$\text{slope} = \frac{3080 - 2948}{44 - 42} = 66$$

Problem 2 (2.1 #4)

(a) (i) $x = 2$ implies $Q = (2, .5)$ so that the slope of the line PQ is

$$\frac{.5 - 2}{2 - .5} = -1$$

The rest are similar:

(ii) $Q = (1, 1)$, so

$$\text{slope} = \frac{1 - 2}{1 - .5} = -2$$

(iii) $Q = (.9, 10/9)$, so

$$\text{slope} = \frac{10/9 - 2}{.9 - .5} \approx -2.222222$$

(iv) $Q = (.8, 1.25)$, so

$$\text{slope} = \frac{1.25 - 2}{.8 - .5} = -2.5$$

(v) $Q = (.7, 10/7)$, so

$$\text{slope} = \frac{10/7 - 2}{.7 - .5} \approx -2.857143$$

(vi) $Q = (.6, 5/3)$, so

$$\text{slope} = \frac{5/3 - 2}{.6 - .5} \approx -3.333333$$

(vii) $Q = (.55, 20/11)$, so

$$\text{slope} = \frac{20/11 - 2}{.55 - .5} \approx -3.636364$$

(viii) $Q = (.51, 100/51)$, so

$$\text{slope} = \frac{100/51 - 2}{.51 - .5} \approx -3.921569$$

(ix) $Q = (.45, 20/9)$, so

$$\text{slope} = \frac{20/9 - 2}{.45 - .5} \approx -4.444444$$

(x) $Q = (.49, 100/49)$, so

$$\text{slope} = \frac{100/49 - 2}{.49 - .5} \approx -4.08$$

(b) It seems that -4 is a reasonable guess at the slope of the tangent line at P.

Problem 3 (2.1 # 6)

(a) Compute the slopes using the formula

$$\text{slope} = \frac{h(t_1) - h(t_0)}{t_1 - t_0}$$

where $h(t) = 58t - .83t^2$.

(i) 55.51
(ii) 59.2865
(iii) 56.257
(iv) 56.3317
(v) 56.33917

(b) The numbers are creeping towards 56.34.

Problem 4 (2.1 # 8)

(a) (i)

$$\frac{178 - 32}{3} \approx 48.7$$

(ii)

$$\frac{119 - 32}{2} \approx 43.5$$

(iii)

$$\frac{70 - 32}{1} = 38$$

(b) Throw a dart.

Problem 5 (2.2 #2)

$$\lim_{x \to 1^-} f(x) = 3$$

means that, as x gets near 1 but remains less than 1, f gets near 3. Similarly,

$$\lim_{x \to 1^+} f(x) = 7$$

means that as x nears 1 and remains greater than 1, f nears 7.

It is certainly not possible that

$$\lim_{x \to 1} f(x)$$

exists in this case, for this would mean that, as x nears 1 from *any* direction, then f nears one value.

Problem 6 (2.2 #4)

(a) 3

(b) 4

(c) 2

(d) does not exist - different left and right limits (as explained in the solution to the previous homework problem)

(e) 3

Problem 7 (2.2 #6)

(a) -1

(b) 1

(c) does not exist - different left and right limits

(d) 1

(e) 1

(f) 2

(g) does not exist - different left and right limits

(h) 2

(i) does not exist - oscillates infinitely often between 1.5 and 2.5 near to and greater than $x = 4$

(j) 4

(k) does not exist - wasn't defined

(l) 0

Problem 8 (2.2 #8)

(a) 0

(b) ∞

(c) $-\infty$

(d) $-\infty$

(e) There are vertical asymptotes with equations $x = -5$, $x = 0$, and $x = 4$.

Problem 9 (2.2 #12)

$$\lim x \to af(x)$$

exists for all a except -1 and 1. Ask in section if your graph doesn't agree with this.

Problem 10 (2.2 #14)

Ask in section.

Problem 11 (2.2 #22)

While x is less than 5, $x - 5$ is negative, so that $6/(x - 5)$ is also negative. Thus the limit is $-\infty$.

Problem 12 (2.2 #26)

Recall that

$$\csc(x) = \frac{1}{\sin(x)}$$

and that $\sin(x)$ is positive immediately to the left of π (where it vanishes), so

$$\lim_{x \to \pi^-} \csc(x) = \infty.$$

Problem 13 (2.2 #28)

While $x \to 5^+$, $x - 5 \to 0^+$, and so

$$\lim_{x \to 5^+} \log(x) = -\infty.$$

If this is unfamiliar, recall the graph of $\log(x)$ (or have me graph it for you in section). In particular, recall that $\log(x)$ is negative for $0 < x < 1$ and that the graph has a vertical asymptote at 0.