Primer Assignment for Mathematics 1b
Problem Set # 0

Part I. Graphing Primer

Learning Goal: There are many different ways to look at mathematical problems; often a graphical approach is fruitful. In order to use this approach successfully, you need familiarilty with the graphs of some basic functions. For example, in addition to being able to graph lines and parabolas, you should have some expectations about what the graphs of higher order polynomials can look like. (This will be important for understanding Taylor and MacLaurin series.) You should be able to draw the graphs of trigonometric functions such as $\sin x$, $\cos x$ and $\tan x$, of exponential functions (such as e^x and e^{-x}), and of the logarithmic function. This is by no means an exhaustive list of all the functions you may run into in your studies, but it is a beginning. Some of you may have become accustomed to leaning very heavily on a graphing calculators for anything having to do with graphing. The exercises that follow are meant to prime your graphing skills. You should use a graphing calculator or (the graphing capacity of a computer) to *check* your work but not to do it.

1. How are the graphs of $y = f(x)$ and $y = f(x-2)$ related? If the zeros (roots) of $f(x)$ are at $x = 3, 7$, and 10, what are the zeros of $f(x-2)$?
2. Look at the graphs on the bottom of the next page. Write a possible formula for each function. (There may be more than one correct answer to some of these problems.) Check your answer with a graphing calculator or a computer.
3. What are characteristics of polynomials that distinguish them from exponential, trigonometric, and logarithmic functions.

Part II. Primer for Taylor Series

The first third of this course deals primarily with series and polynomial approxiations of functions; the second third deals with integration and its applications. You already should know how to find the equation of the tangent line to a differentiable curve at a given point and should be able top interpret an integral as the area under a curve. In this set of questions you'll review this material and start to see how Taylor polynomials are constructed.

Suppose we want to approximate $\ln 1.1$.

1. The natural logarithm can be defined as an integral: $\ln x = \int_1^x \frac{1}{t}\, dt$.
   (a) Sketch the graph of $y = \frac{1}{t}$ and shade the area under the graph that corresponds to $\ln 1.1$.
   (b) Find an upper bound for $\ln 1.1$ by approximating the shaded area with the area of a circumscribed rectangle.
   (c) Find a lower bound for $\ln 1.1$ by approximating the shaded area with the area of an inscribed rectangle.
   (d) Now approximate $\ln 1.1$ by approximating the shaded area with the area of a trapezoid. (Notice that this is simply the average of the answers to (b) and (c).)
   (e) Suppose you wanted a greater degree of accuracy. Describe what you could do to approximate the shaded area more accurately. (You need not do any computations for this part of the question.)

2. Tangent line approximation:
   (a) Find the equation of the line tangent to $f(x) = \ln(1+x)$ at the point $x = 0$.
   (b) On the same set of axes, sketch the graph of $f(x) = \ln(1+x)$ and the line you found in part (a).
   (c) Use your tangent line approximation to $f(x) = \ln(1+x)$ to approximate $\ln 1.1$. (Let $x = 0.1$.) From your picture in part (b), do you expect your approximation to be too large, or to be too small?
   (d) Compare your approximations of $\ln 1.1$ from problems 1 and 2 with the value given by your calculator or a computer.

   It is possible to refine the approaches taken in both problems 1 and 2 to improve the approximations. In the next problem, we'll improve upon the tangent line approximation.

3. We know that the tangent line to a function at a point is the best linear approximation to that function at that point. We can improve upon the tangent line approximation by finding the quadratic of best fit at $x = 0$.

   (a) Show that the quadratic $y = x - \frac{x^2}{2}$ intersects the graph of $f(x) = \ln(1 + x)$ at $x = 0$, that it has the same first derivative as $f(x) = \ln(1 + x)$ at $x = 0$, and that it has the same second derivative as $f(x) = \ln(1 + x)$ at $x = 0$.

   (b) On the same set of axes, graph $f(x) = \ln(1 + x)$, the tangent line approximation to $f(x) = \ln(1 + x)$ at $x = 0$ and the quadratic approximation $y = x - \frac{x^2}{2}$. Zoom in around $x = 0$ and record your observations.

   (c) Compare the values of $f(x) = \ln(1 + x)$ and $x - \frac{x^2}{2}$ at $x = .1$