Name: ______________________________

Second Examination

Mathematics 1b

Fall, 2001

Problem	Points	Score
1	15	
2	13	
3	12	
4	10	
5	11	
6	10	
7	10	
8	10	
9	9	
Total	100	

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get credit unless otherwise indicated. If you have work on the back of a page, indicate that on the exam cover.

You have two hours for this exam. Work carefully and efficiently. Do not spend an inordinate amount of time on any one problem. No calculators allowed. Think clearly and do well!

Please circle your section.

MWF 9	David Jao	MWF 10	Daniel Allcock	MWF 10	Robin Gottlieb
MWF 11	Albert Chau	MWF 11	Robin Gottlieb	MWF 12	Stephen Debacker
TTH 10	Huan Yang	TTH 10	Pete Clark	TTH 10	Alina Marian
TTH 11:30	Huan Yang	TTH 11:30	Dmitry Tamarkin		

Mathematics 1b Second Examination

1. (15 points) You need not show your work on this question.

(i) Which one of the following are solutions to the differential equation $\frac{dw}{ds} = (w-2)^2(w+1)$? On the blank line below, indicate the letter corresponding to the correct graph

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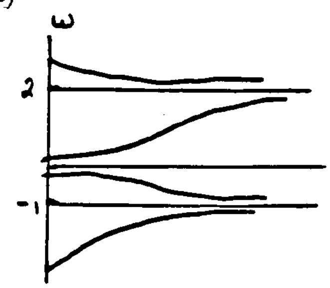
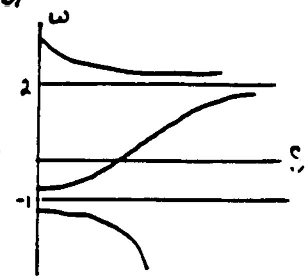
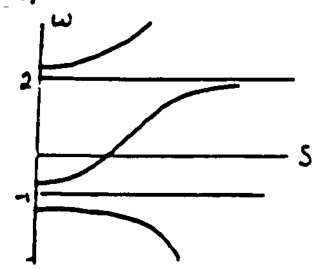
(a)



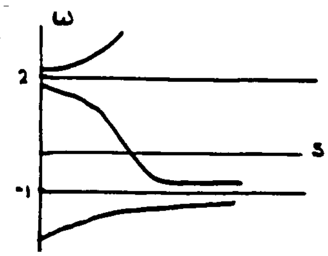
b)



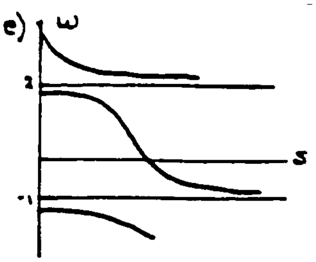
c)



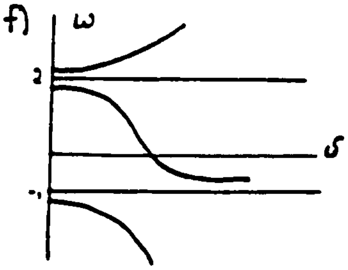
d)



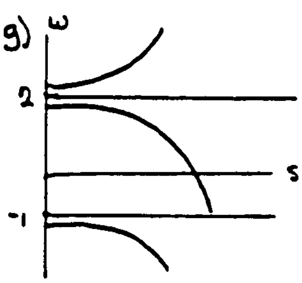
e)



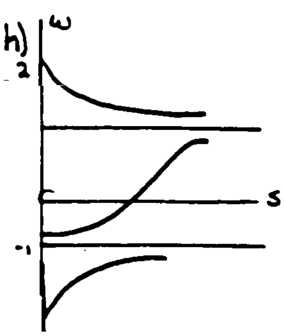
f)



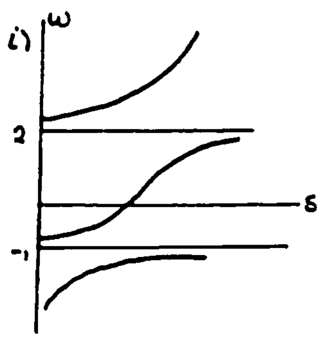
g)



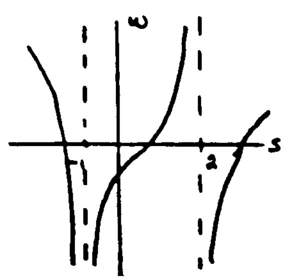
h)



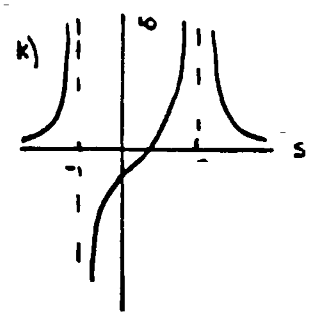
i)



j)



k)

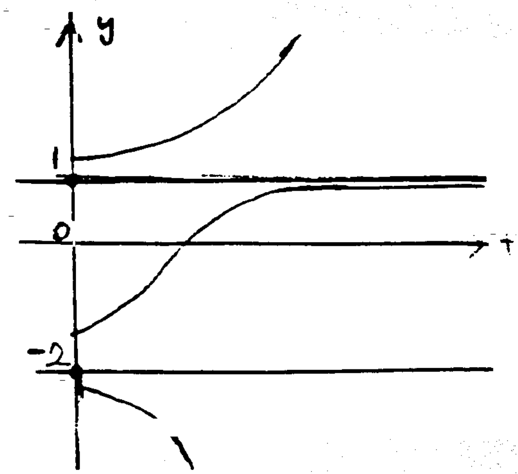


(ii) Which *one* of the following differential equations has a **stable** equilibrium at $P = 5$?

(a) $\frac{dP}{dt} = 5$ (b) $\frac{dP}{dt} = P - 5$ (c) $\frac{dP}{dt} = (P-5)^2$

(d) $\frac{dP}{dt} = 25 - P^2$ (e) $\frac{dP}{dt} = t^2 - 25$ (f) $\frac{dP}{dt} = (P-1)(P-4)$

(iii) Below is a graph of several particular solutions to a differential equation.



Which one of the following differential equations could have the solutions pictured above?

(a) $\frac{dy}{dt} = (y-1)(y+2)$ (b) $\frac{dy}{dt} = (1-y)(y+2)$ (c) $\frac{dy}{dt} = (y-1)^2(y+2)$

(d) $\frac{dy}{dt} = (y-1)(y+2)^2$ (e) $\frac{dy}{dt} = -(y-1)^2(y+2)$ (f) $\frac{dy}{dt} = -(y-1)(y+2)^2$

2. (13 points)

(a) Graph $f(x) = \frac{1}{x^2}$ and then determine whether or not the integral

$$\int_{-2}^{2} \frac{dx}{x^2}$$

converges. If the integral does converge, evaluate it.

(b) Suppose that an arbitrary function $f(x)$ has the following characteristics:

- $f(x)$ is continuous and positive on $(-\infty, \infty)$
- $\lim_{x\to\infty} f(x) = 0$

Which of the following statements is true?

i. $\int_1^\infty f(x)\,dx$ definitely converges.

ii. $\int_1^\infty f(x)\,dx$ definitely diverges.

iii. We don't have enough information to determine whether or not $\int_1^\infty f(x)\,dx$ converges.

Explain:

3. (12 points) In this problem we ask you to write an integral giving the volume for each of the solids described. You need *not* evaluate the integrals, just write them.

   Let R be the area under the curve $y = \sin(x)$ between $x = 0$ and $x = \pi/2$

   (a) Write an integral that gives the volume of the solid of revolution obtained by rotating R around the vertical line $x = -2$.

   (b) Write an integral that gives the volume of the solid of revolution obtained by rotating R around the horizontal line $y = 3$.

4. (10 points) Let $P = P(t)$ be the number of rabbits in a rabbit colony at time t. In isolation the rabbit population would grow at a rate proportional to itself with proportionality constant 0.1 per year. But in fact the rabbits do not live in isolation and are being eaten by predators at a rate of $10^{-7}P^3$ per year.

   (a) Write a differential equation modeling the situation and satisfied by $P(t)$.

   (b) What are the equilibrium solutions of the differential equation in part (a)?

   (c) When there are 100 rabbits, at what rate is the rabbit population changing? (Please simplify your answer.)

5. (11 points) The world's largest helicopter is the Soviet Mi-26 Halo. Suppose that an Mi-26 takes off vertically from the ground and rises to a height of 2000 meters by traveling at a constant speed of 1 meter per second. At take-off the helicopter and its gas tank have an initial combined mass of 56,000 kg. During its rise the helicopter consumes 1 kg of fuel per second. (It is getting lighter as it rises.) What is the total work done lifting the helicopter from the ground to 2000 meters? *You can leave your answer as a product and difference of numbers without explicitly doing the multiplication and subtraction.*

   Note: Force = m $\cdot g$ where m is mass and $g = 9.8$ m/sec^2.

6. (10 points) Solve the differential equation below given the initial condition $y(0) = 0$.

$$\frac{dy}{dx} = x^2 e^{y-x^3}$$

   Please solve for y explicitly in terms of x.

7. (10 points) A cylindrical sewage pipe full of muddy water is lying horizontally on the ground. The density of the mud in the pipe is given by $\rho(z)$ grams/cm^3 where z is the distance from the ground. The pipe is 3000 cm long and 100 cm in diameter. Which of the following gives the mass of the mud in the pipe?

Explain your reasoning by drawing a picture of how you are slicing and describing a generic slice in words and a picture. Explain where the components of the integrand come from.

(a) $\displaystyle\int_0^{50} 2\pi z \cdot \rho(z) 3000\, dz$

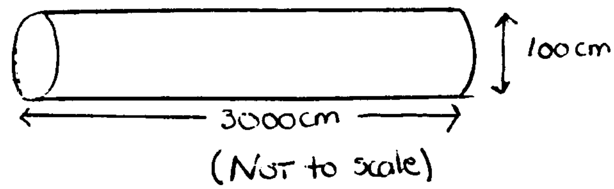
(b) $\displaystyle\int_0^{50} 2\pi z^2 \cdot \rho(z) 3000\, dz$

(c) $\displaystyle\int_0^{100} 2\sqrt{50^2 - (50-z)^2} \cdot \rho(z) 3000\, dz$

(d) $\displaystyle\int_0^{100} 2\pi z \cdot \rho(z) 3000\, dz$

(e) $\displaystyle\int_0^{3000} 2\pi z 50 \cdot \rho(z) 3000\, dz$

(f) $\displaystyle\int_0^{100} (50 - z) \cdot \rho(z) 3000\, dz$



Explanation:

8. (10 points) Consider the function $f(x) = \frac{C}{x^2+1}$ on the domain $(-\infty, \infty)$, where C is a constant. If C is chosen correctly, then f is a probability density function.

a) Find the value of C such that f is a probability density function. b) What is the probability that $X \geq 0$?

c) The probability that $0 \leq X \leq \sqrt{3}$ is $\frac{1}{3}$. What is the probability that $X \geq \sqrt{3}$?

9. (9 points) Let $f(x)$ be a continous function that is positive, decreasing, and concave up on the interval $[a, b]$. Let $I = \int_a^b f(x)\,dx$.

Your job is to order I and the numerical approximations given by L_{15}, R_{15}, M_{15}, and T_{15} where L_{15}, R_{15}, M_{15}, and T_{15} (left-hand, right-hand, midpoint, and trapezoidal sums) are defined as follows:

Partition $[a, b]$ into 15 equal subintervals, each of length Δx. Let $x_k = a + \Delta x$ for $k = 0, 1, 2, \ldots, 15$.

- $L_{15} = \displaystyle\sum_{k=1}^{15} f(x_{k-1})\Delta x$
- $R_{15} = \displaystyle\sum_{k=1}^{15} f(x_k)\Delta x$
- $M_{15} = \displaystyle\sum_{k=1}^{15} f\left(\frac{x_{k-1} + x_k}{2}\right)\Delta x$
- $T_{15} = \dfrac{L_{15} + R_{15}}{2}$

Put I, L_{15}, R_{15}, M_{15}, and T_{15} in ascending order (with '$<$' signs between them). Explain your reasoning briefly but precisely.