Handout A

1. Determine whether or not the following sums are geometric. If the sum is geometric, express the sum in closed form.

   a) $\sum_{k=1}^{70}(\frac{1}{k})$ b) $\sum_{k=1}^{50}(\frac{1}{k})^2$ c) $\sum_{k=1}^{60}(\frac{1}{k})^k$ d) $\sum_{k=1}^{60}(1.01)^{k/12}$

2. Find the sum of each of the following.

   a) $\sum_{k=0}^{100}(\frac{1}{3})^k$ b) $\sum_{k=0}^{\infty}(\frac{1}{3})^k$ c) $\sum_{k=2}^{100}(\frac{1}{3})^k$ d) $\sum_{k=2}^{\infty}(\frac{1}{3})^k$

3. Question: Do the first few terms of the series affect whether or not the series converges? Do the first few terms of a convergent series affect its sum?

   think, and think again problems. Here is the message about such problems for the student.

   You will receive full credit on your homework for a thoughtful answer to such a question - regardless of whether you have answered it correctly. You need to be prepared to discuss your answer in class - that is, to think about it at home and then to think about it again in class.

   Think, and think again (#1):

   1. Suppose you know that the infinite series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ converges and that $a_k > 0$ for k any positive integer. Let $s_n = a_1 + a_2 + a_3 + \cdots + a_n$. For each of the following statements, determine whether the statement must be true, could possibly be true, or must be false.

      (a) $\lim_{n\to\infty} a_n = 0$

      (b) $\lim_{n\to\infty} s_n = 0$

      (c) There exists a number M such that $s_n < M$ for all n.
      (This is equivalent to saying that the partial sums are bounded.)

      (d) $\sum_{k=5}^{\infty} a_k$ converges

   2. Suppose you know that $\lim_{n\to\infty} b_n = 0$. Can you be sure that the infinite series $b_1 + b_2 + b_3 + \cdots + b_n + \cdots$ converges?

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   Think, and think again (#2): Write out the first few terms of the series $\sum_{k=1}^{\infty}\frac{1}{2^k+k}$. (This series is not a geometric series.) Now write out the first few terms of the geometric series $\sum_{k=1}^{\infty}\frac{1}{2^k}$. By comparing the terms of the two series, determine whether or not the former series converges. Explain your reasoning in words carefully and clearly. Your answer will form the launching pad for the next class.

4. *Problem on $p-$series*

   In this problem you will learn about a family of series known as $p-$series. This is an important problem, and we will use the result of your work repeatedly both in class and on exams.

   A $p-$series is a series of the form

   $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots.$

   (a) If $p < 0$ then the series diverges by the nth term test. Explain.

   (b) Show that if $p > 1$ then $\int_1^{\infty}\frac{1}{x^p}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^p}\,dx$ is finite.
   Show that if $p = 1$ then $\lim_{b\to\infty}\int_1^b \frac{1}{x^p}\,dx = \infty$.
   Show that if $0 < p < 1$ then $\lim_{b\to\infty}\int_1^b \frac{1}{x^p}\,dx = \infty$.
   Conclude that $\int_1^{\infty}\frac{1}{x^p}\,dx$ diverges for $0 < p < 1$ and converges for $p > 1$.

   (c) Conclude from your work in parts (a) and (b) that $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$ converges if $p > 1$ and diverges if $p < 1$.