Additional Problems for Mathematics 1b
Handout B

1. The Alternating Series Test says that if an infinite series is

   i) alternating
   ii) the magnitude of the terms is decreasing
   iii) the magnitude of the terms tends to zero

   then the series converges.

   If (iii) is not satisfied then the series diverges by the Nth term test for Divergence. If any one of the other two conditions is not satisfied then the test is INCONCLUSIVE - meaning the series might converge or it might diverge.

   In each of the following situations, find a series that diverges:

   (a) (i) and (ii) are true but (iii) is not

   (b) (ii) and (iii) are true but (i) is not

   Below (for your reading pleasure) we present a series that diverges and (i) and (iii) are satisfied but (ii) is not. (We're doing this case for you because it is harder than the previous two.)

   Conditions (i) and (iii) are satisfied by the series

   $$1 - \frac{1}{2} + \frac{2}{2} - \frac{1}{3} + \frac{2}{3} - \frac{1}{4} + \frac{2}{4} - \cdots + \frac{2}{n} - \frac{1}{n} + \cdots$$

   but

   $$1 + \left(-\frac{1}{2} + \frac{2}{2}\right) + \left(-\frac{1}{3} + \frac{2}{3}\right) + \left(-\frac{1}{4} + \frac{2}{4}\right) + \cdots + \left(\frac{2}{n} - \frac{1}{n}\right) + \cdots$$

   can be written

   $$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \frac{1}{n} + \cdots$$

   which is the harmonic series. The harmonic series diverges, so the series displayed diverges.

2. What exactly do we mean when we write $\sum_{k=0}^{\infty} a_k = 4$? Your answer can be brief, but must be precise and accurate. You will get full credit only if you use words correctly.

3. Suppose that a power series of the form $\sum_{k=0}^{\infty} c_k(x-2)^k$ has a radius of convergence of 7. What are the possibilities for the interval of convergence of the series?

4. Suppose the interval of convergence of the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ is $(-5, 7]$.

   (a) What is a?
   (b) Does the series converge for $x = 6.5$? For $x = -6.5$?