1. Show that $\int_0^3 \sqrt{9-x^2}\,dx = \frac{1}{4}9\pi$ - in other words, show analytically that the area of a circle of radius 3 is 9π by doing the following:

   We'd like to eliminate $\sqrt{9-x^2}$ by making a substitution that makes the integrand a perfect square. We will exploit the trig identity $\sin^2 t + \cos^2 t = 1$, or, equivalently, $9\sin^2 t + 9\cos^2 t = 9$ We know that $9 - 9\sin^2 t$ is a perfect square, so we'll use the substitution $x = 3\sin t$. Now we need to write the entire integral in terms of t.

   a) If $x = 3\sin t$ then what is dx in terms of t?
   b) If $x = 3\sin t$ then what is $\sqrt{9-x^2}$ in terms of t?
   c) If $x = 3\sin t$ then what are the new endpoints of integration in terms of t?
   d) Write the integral in terms of t.
   e) Evaluate the integral in (d).
   f) Conclude that the area of a circle of radius 3 is 9π.

2. Find $\int \sin^2\theta\,d\theta$. Hint: the identities

$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta) \quad \text{and} \quad \sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

   can be useful in integrating $\int \sin^2\theta\,d\theta$ and $\int \cos^2\theta\,d\theta$. In this case the former is simplest. (If you forget these formulas you can use parts to do the integrals. You might want to check this out on your own.)

   For the next four problems, evaluate the integrals. Not all require trigonometric substitution. (In fact, at least two of the four do not. First see if straightforward substitution works.

3. $\int \frac{x}{\sqrt{4+x^2}}\,dx$

4. $\int_0^1 \sqrt{2-t^2}\,dt$

5. $\int_0^1 x^3\sqrt{4-x^2}\,dx$

6. $\int_0^1 \frac{x^3}{\sqrt{9-x^2}}\,dx$

7. Show that the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive, is given by πab. (Recall that in a previous homework you've already shown that the area is between $2ab$ and $4ab$.) (This uses a technique learned in another class; it is not a question using the techniques of §5.9.)

   (For extra credit try p. 441 #72 or the following problem:

8. a) Consider Simpson's Rule for $n = 3$ so that $a_0 = a$, $a_1 = (a+b)/2$, $a_2 = b$ and Simpson's Rule says that

$$\int_a^b f(x)dx \approx \frac{(b-a)}{6}\left[f(a) + 4f(\frac{a+b}{2}) + f(b)\right].$$

   Use this formula for $f(x) = 1$, $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$and check that in each case it gives the exact value of the integral.

   b) Let $f(x) = e + fx + gx^2 + hx^3$, where e, f, g, h are numbers. Without computation, only basing on a), find out whether for such a function Simpson's Rule gives an exact value of the integral.