Handout G

Feeling Blue, Looking Bluer

Spraying a piece of pottery with cobalt will result in a blue color when the piece is fired. The shade of blue is determined by the density of cobalt; the greater the density of the cobalt application, the darker the blue of the pot. You can get gradations of blue by applying cobalt glaze with a spray gun and varying the density of the application. Makoto and Wasma are professional potters at the Radcliffe Pottery Studio on Western Avenue.

1. Makoto has made a rectangular sushi platter from a slab of clay 14 inches by 6 inches. He applies cobalt such that the density of the application increases with the distance from one of the long sides of the platter. The density of cobalt glaze is given by $\rho(x)$ mg/square inch where x is the distance (in inches) from one long side of the sushi platter.

   a) How can you approximate the amount of cobalt Makoto used?

   b) Give an expression in terms of $\rho(x)$ that gives the amount of cobalt used.

2. Makoto decides to try a more symmetric glaze application on his next sushi platter. The platter is again 14 inches by 6 inches. This time the deepest blue is in a stripe along the long center line of the platter and the intensity of the blue fades with the distance from this central line. The density of cobalt glaze is given by $\rho(x)$ mg/square inch where x is the distance (in inches) from the longitudinal center of the sushi platter.

   a) How can you approximate the amount of cobalt Makoto used?

   b) Give an expression in terms of $\rho(x)$ that gives the amount of cobalt used.

3. Wasma is glazing a large round plate 16 inches in diameter. She decides to have a deep blue center fading out into pale blue along the rim. She applies cobalt glaze such that its density is given by $\rho(x)$ mg/square inch where x is the distance (in inches) from the center of the plate.

   a) How can we approximate the amount of cobalt Wasma used?

   b) Give an expression in terms of $\rho(x)$ that gives the amount of cobalt used.

4. *Leaving the Pottery Studio and Entering the Wild World*

   Between December and July the Serengeti in Tanzania is the scene of a mass animal migration as over 1 million wildebeest, 200,000 zebra and 300,000 Thomson's gazelle journey across the plains in search of new grazing lands and water. Suppose $f(x)$ gives the rate at which zebra are entering/leaving the Seronera region of the Serengeti, where $f(x)$ is given in tens of thousands of zebra per month and $t = 0$ corresponds to January. How can we interpret $\int_{-1}^{1} f(x)\, dx$?

5. The density of a ball of ice is greatest at the center and decreases with the distance from the center of the ball. The ball is 10 centimeters in radius and the density is given by $\rho(x)$ grams per square centimeter. What is the mass of the ball?