More Problems

1. Rocket fuel is stored in a hemispherical tank of radius 5 m. Rocket fuel has a density of 100 kg/m^3. How much work is done if the tank is emptied out via a pipe located 2 m above the tank? (See Figure 1 below.)

2. *Surface area of a surface of revolution*

   Suppose $0 < a < b$ and f is a continuous non-negative function on $[a, b]$. Let S be the surface obtained by revolving the curve $y = f(x)$ about the x-axis. (See Figure 2 below.) The surface area of S is *defined* to be

   $$\int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx. \qquad (*)$$

   The point of this exercise is to provide a mathematical justification for this definition.

   (a) A frustum with a slant height l, larger radius R and smaller radius r has surface area

   $$\pi(r + R)l.$$

   (See Figure 3 below.)

   Explain (in less than 30 words) why this formula is 'reasonable' .

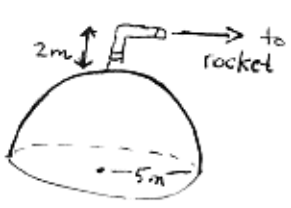
   (b) Show that

   $$l = h\sqrt{1 + \left(\frac{R - r}{h}\right)^2}.$$

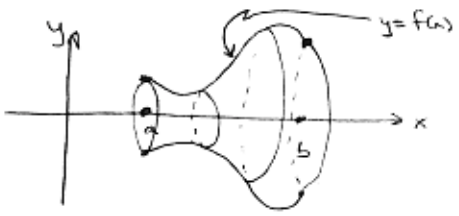
   (c) By making appropriate slices, show that the contributions of the ith slice to the surface area is S is approximately

   $$2\pi f(x_i)\sqrt{1 + (f'(x_i))^2}\, \Delta x.$$

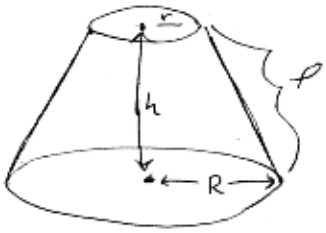
   (d) Use your answer to (c) to derive (*).



1.



2.



3.