# *Mathematics 1b*
## Handout J

1. Consider the differential equation

$$\frac{dy}{dx} = y - x + 1.$$

Suppose $y(0) = 1$.

(a) What is the slope of the solution curve passing through (0, 1) at the point (0,1)?

(b) Use Euler's method to estimate $y(1)$ in one step.

(c) Same as (b), but now estimate $y(1)$ in two steps, by first estimating $y(0.5)$.

(d) Verify that $y = e^x + x$ is a solution to the differential equation that satisfies the initial condition $y(0) = 1$, and compare the true value of $y$ at $x = 1$ to the values you got in parts (b) and (c).

2. Suppose $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = x^2 + y^2$ and passes through the point $(x, y) = (1, 1)$.

(a) What is the slope of the curve $y = f(x)$ at $(x, y) = (1, 1)$?

(b) Make a rough sketch of the graph of $y = f(x)$.

(c) Give the equation of the tangent line at $(x, y) = (1, 1)$.

(d) Is the graph of $y = f(x)$ concave up or concave down at $(x, y) = (1, 1)$? Explain.

3. Describe the behavior of the solution of the differential equation

$$\frac{dy}{dt} = (y - 3)^2$$

with initial condition $y(0) = 4$, as $t$ gets larger.

4. Suppose $y$ satisfies the differential equation $\frac{dy}{dx} = y^2$.

(a) Describe the behavior of $y$ assuming each one of the following initial conditions

(i) $y(0) = 0$, (ii) $y(0) = 0.01$ (iii) $y(0) = -0.01$.

(b) What kind of equilibrium (stable versus unstable) is the solution you found in 4.a(i)?

(c) Using separation of variables, explicitly solve the differential equation $\frac{dy}{dx} = y^2$ with initial condition $y(0) = 1$.

(d) What happens to $y$ as $x$ goes to 1?

Note that from the work you did in this problem you can deduce that the solution in #3 not only increases without bound, but increases without bound in finite time! A similar thing happens with the differential equation $\frac{dy}{dx} = (y-1)(y-3)$ that we studied on pages 11-12 of the Supplement under appropriate initial conditions.

5. Didi X., a student in Math 1b, comes to her classmate Laurin Mactaylor for some help.

   (a) "I don't see why I got this one wrong," says Didi. "They say to let $y(t)$ be the amount of dioxin in the ground underneath Mather House, and to assume that it goes down by 100 gallons a year, so I wrote $y' = -100t$. Why isn't this right?" How should Laurin explain to Didi what parts of her answer were right and what parts were wrong?

   (b) "Here's another one that the TA marked off without explaining why," says Didi. "The problem said that the property that the Lampoon sits on has exponentially increasing value over time, so I wrote down the differential equation $y' = y_0 e^{kt}$, where $y_0$ and $k$ are constants to be determined later. But the TA marked this as wrong, even before I tried to solve for $y_0$ and $k$." Laurin recognizes that Didi is mixing up two true equations (one of which has $y'$ on the left and the other of which has $y_0 e^{kt}$ on the right). How should Laurin explain to Didi what two things she's mixing up?

   (c) What seems to be Didi's underlying confusion, and how might Laurin address this issue?