Handout K

1. A canister contains 10 liters of blue paint. Paint is being used at a rate of 2 liters per hour and the canister is being replenished by pale blue paint that is 80% blue paint and 20% percent white paint. Asume the canister is well-mixed and that paint is both entering and leaving the canister continuously.

   a) Write a differential equation whose solution is $w(t)$, the amount of white paint in the canister at time t. Specify the initial condition. Use qualitative analysis (and common sense) to sketch the solution to the differential equation.

   b) Write a differential equation whose solution is $b(t)$, the amount of blue paint in the canister at time t. Specify the initial condition. Use qualitative analysis (and common sense) to sketch the solution to the differential equation.

2. Let $x = x(t)$ be the number of thousands of beasts of species X at time t. Let $y = y(t)$ be the number of thousands of beasts of species Y at time t. Suppose that

$$\frac{dx}{dt} = .1x - .05xy$$

$$\frac{dy}{dt} = .1y - .05xy$$

   a) Is the interaction between species X and Y symbiotic, competitive, or a predator/prey relationship?

   b) What are the equilibrium populations?

   c) Find the nullclines and draw directed horizontal and vertical tangents in the phase plane. (Consult the supplement for illustrations.)

   d) The nullclines divide the first quadrant of the phase plane into four regions. In ease region, determine the general directions of the trajectories.

   e) If $x = 0$, what happens to $y(t)$? How is this indicated in the phase plane?
   If $y = 0$, what happens to $x(t)$? How is this indicated in the phase plane?

   f) Use the information gathered in parts (b)-(e) to sketch representative trajectories in the phase plane. Include arrows indicating the direction in which the trajectories are travelled.

   g) For each of the initial conditions given below, describe how the size of the populations of X and Y change with time and what the situations will look like in the long run.

i. $x(0) = 2$ $y(0) = 1.8$
ii. $x(0) = 2$ $y(0) = 2.3$
iii. $x(0) = 2.2$ $y(0) = 2$

h) Does this particular model support or challenge Charles Darwin's Principle of Competitive Exclusion?

3. Consider the following model for the population levels of predators and prey, in a given enviroment at time t.

$$\frac{dx}{dt} = ax - bx^2 - cxy$$

$$\frac{dy}{dt} = -dy + exy$$

All the parameters a, b, c, d, e are positive constants.

(a) Which of x and y are the predators (and prey)? Explain please.

(b) If you started of with some prey and no predators, what do you expect in the long run?

Suppose, on the other hand, you start with some predators and no prey. What happens in the long run?

(c) Show that there is an equilibrium for the system at the point $(x, y) = \left(\frac{d}{e}, \frac{a}{c} - \frac{bd}{ce}\right)$.

(d) Give some interpretation to the parameters $a, b, c, d,$ and e.

4. Consider the following model for the populations of two species in competition in a given enviroment at time t (say,.. mice and rats).

$$\frac{dx}{dt} = x - x^2 - axy$$

$$\frac{dy}{dt} = y - y^2 - axy$$

The parameter a is a positive constant.

(a) If you started out with some mice, no rats, what would you expect to see in the long run? (What about some rats, no mice)

(b) Sketch the phase portrait (label the equilibrium points and null clines) of the system for the cases $a = 1/2$ and $a = 3/2$ and note the difference in the behavior of the system. Please describe the difference between the cases $a < 1$ and $a > 1$.