# Handout L

Consider a population $A$ of aphids and $L$ of ladybugs in a large broccoli patch.

1. If the aphids are alone, they grow according to the differential equation

$$\frac{dA}{dt} = A(100,000 - A).$$

   Suppose we start with 100 aphids. What happens in the long run? Sketch a rough graph showing $A$ as a function of time. (You need not put units on the $t$axis, but you should label important value(s) on the $A$ axis.)

2. If the ladybugs are alone, $L$ satisfies

$$\frac{dL}{dt} = L(-1000 - L).$$

   Suppose we start with 100 ladybugs. What happens in the long run? Sketch a rough graph showing $L$ as a function of time. (You need not put units on the $t$axis, but you should label important value(s) on the $L$ axis.)

3. When the aphids and ladybugs are together, the populations satisfy

$$\frac{dA}{dt} = A(100,000 - A - 50L).$$

$$\frac{dL}{dt} = L(-1000 - L + \frac{1}{25}A).$$

   Do a qualitative phase plane analysis for these differential equations showing the equilibrium point(s), the null clines, and the direction of the trajectories in each region.

   Note: In fact, the trajectories spiral in. Make sure your results are consistent with this.