Looking at Integration from a Graphical Perspective due Wed, 03/13, or Thursday, 03/14

This worksheet encourages you to make estimations, use symmetry, and, in general, take a graphical look at definite integrals. You are welcome to do the worksheet on your own – but if you'd like to work as a group, the Course Assistants will facilitate this during their problem sessions this week.

For each of the following, use an appropriate graph to evaluate the truth of falsehood of each claim. (Graphing calculators or computers can produce graphs for you.) Your answers should include pictures.

- Claim 1. $0 < \int_0^a e^{-x^2}\,dx < a$
- Claim 2. $\int_0^{\sqrt{\pi}} \cos(x^2)\,dx < 0$
- Claim 3. $\int_{-\pi}^{\pi} e^{-x^2/\sqrt{2}}\,dx = 2\int_0^{\pi} e^{-x^2/\sqrt{2}}\,dx$
- Claim 4. $\int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx < \int_1^2 \frac{1}{\sqrt{1+x^4}}\,dx$
- Claim 5. $\int_{-3}^{3} \frac{x}{1+x^4}\,dx > 0.001$
- Claim 6. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is greater than $2ab$ and less than $4ab$.

  plus:

  On the time interval $[a, b]$ a car's velocity, $v(t)$, is positive and increasing. The velocity is increasing at a decreasing rate on this interval. Suppose we partition the interval $[a, b]$ into 10 equal subintervals, each of length Δt. Let $t_k = a + k\Delta t$ where $k = 0, 1, 2, \cdots 10$.

- Claim 7. $\sum_{k=1}^{10} v(t_{k-1})\Delta t >$ the distance travelled on $[a, b]$.
- Claim 8. $\sum_{k=1}^{10} v(t_k)\Delta t >$the distance travelled on $[a, b]$.
- Claim 9. $\frac{1}{2}\left[\sum_{k=1}^{10} v(t_k)\Delta t + \sum_{k=1}^{10} v(t_{k-1})\Delta t\right] <$the distance travelled on $[a, b]$.