

## Solutions to Integration Problems

March 15, 2002

1.  $\frac{1}{2}\sin^{-1}(e^x) + \frac{1}{2}e^x\sqrt{1 - e^{x^2}} + C$
2.  $x \tan x + \ln |\cos x| + C$
3.  $2x + \ln |2x + 1| + C$
4.  $\frac{7}{2}e^{x^2} + C$
5.  $-\frac{1}{x+3} + C$
6. 0
7.  $\frac{2}{9}(31\sqrt{31} - 7\sqrt{7})$
8.  $\frac{1}{2}e^x(\sin x - \cos x) + C$
9.  $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$
10.  $\frac{1}{2}x - \frac{1}{28}\sin(14x) + C$
11.  $\ln |x^4 + x^3 + x^2 + x + 1| + C$
12.  $\frac{9}{2}\ln|x - 1| - 15\ln|x - 2| - \frac{21}{2}\ln|x - 3| + C$
13.  $\frac{1}{2}\sin(x^2) + C$  (substit.)
14.  $x^2 \sin x + 2x \cos x - 2 \sin x + C$  (parts twice)
15.  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(4n+1)!(2n)!}$
16.  $\frac{1}{2}x + \frac{1}{4}\sin(2x) + C$  Use trig identity  $\cos^2 x = \frac{1}{2}[1 + \cos(2x)]$
17.  $\frac{1}{2}x^2 \sin(x^2) + \frac{1}{2}\cos(x^2) + C$
18.  $\sin(x) - \frac{1}{3}\sin^3(x) + C$
19. 1
20.  $x(\ln x)^2 - 2x \ln x + 2x + C$
21.  $\frac{1}{3}(\ln x)^3 + C$
22.  $\ln 3$
23.  $\frac{1}{2}(x^2 + 1)\ln(x^2 + 1) - \frac{1}{2}(x^2 + 1) + C$  Your answer may differ by a constant, of course.
24.  $x - 2 \arctan x + C$
25.  $x - \ln|x + 1| + \ln|x - 1| + C$  Note  $1 + \frac{2}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}$
26.  $\frac{1}{2}\tan^{-1}(x^2) + C$
27.  $\sqrt{x^2 + 1} + C$
28.  $\frac{1}{9}\sqrt{9x^2 - 4} + C$
29.  $\frac{1}{4}\ln|e^x + 2| - \frac{1}{4}\ln|e^x - 2| + C$  Substit followed by partial fractions
30.  $\frac{1}{2}\ln|e^{2x} - 4| + C$  substit.
31.  $2\sqrt{7} - 2\sqrt{5}$
32.  $2\ln|x - 3| - \ln|x + 1| + C$
33.  $x + 2 \tan^{-1} x + C$
34.  $2 \arcsin(x/2) + \frac{1}{2}x\sqrt{4 - x^2} + C$
35.  $x \arctan(5x) - 10 \ln(1 + 25x^2) + C$
36.  $\frac{1}{3}e^{x^3} + C$
37.  $\frac{5}{6}\sin^2(3x) + C$
38.  $-\frac{3}{2}\ln(\cos 2x) + C$
39.  $e^{\tan x} + C$
40.  $-\frac{\cos(2 \sin x)}{2} + C$
41.  $\frac{\ln(1+x)}{(1+x)} + \frac{1}{(1+x)} + C$
42.  $\frac{x^2}{2} - x + 3\ln|1 + x| - 1/(1 + x) + C$
43.  $-\frac{2}{1+x} + C$
44.  $\frac{1}{2}\ln \frac{x}{x+1} + C$
45.  $\frac{x}{9\sqrt{1-9x^2}} - \frac{1}{27}\sin^{-1}(3x) + C$
46.  $3x \sin x + 3 \cos x + C$
47.  $\frac{1}{3}(x^2 + 2x)^{\frac{3}{2}} + C$
48.  $e^{3x} \left[ \frac{2 \sin(2x) + 3 \cos(2x)}{13} \right] + C$
49.  $1 - \cos(1)$
50.  $\frac{x}{2}(\sin(\ln(x)) - \cos(\ln(x))) + C$
51.  $2e^2 + 2$
52.  $\frac{-1}{3x} - \frac{\arctan(x/3)}{27}$
53.  $\frac{x^2}{2} - \frac{1}{2}\ln(x^2 + 1) + C$

54.  $\ln(x) - \frac{1}{2}\ln(x^2 + 1) + C$
55.  $\frac{1}{3}\ln\left|\frac{1-\cos(x)}{4-\cos(x)}\right| + C$
56.  $\ln|e^x - 1| - x + C$
57.  $x^3/3 - x^2/2 + 5x + 4\ln|x - 2| - \ln|x + 1| + C$
58.  $\arcsin(1/2)$
59.  $2e^{\sqrt{x}}$
60.  $\frac{x}{2}\sqrt{9 - 4x^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right) + C$
61.  $\frac{1}{2}\ln(x^2 + 1) + \tan^{-1}(x)$
62.  $e^{(x+1)^2}$
63.  $\frac{\sin(2x)}{2}$
64.  $-\sqrt{1 - x^2}$
65.  $-\frac{1}{2}\cos(x^2 - 1)$
66.  $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$
67.  $\frac{(x^2+1)^5}{5} + \frac{2(x^2+1)^3}{3} + x^2 + 1$
68. 0
69.  $2e^3/9 + 1/9$
70.  $-\frac{1}{4}\cos(2x) + C$
71.  $\ln\ln\ln x + C$
72.  $x(1 - x^2)^{-1/2} + C$
73.  $-\frac{1}{2}\cos 2x - \frac{1}{6}\cos^3 2x + C$
74.  $x \ln \sqrt{x} - x/2 + C$  (begin with the substitution  $x = u^2$ .)
75.  $\frac{2}{4}\ln|x - 2| + \frac{1}{4}\ln|x + 2| + C$
76.  $\frac{x}{\sqrt{1+x^2}}$
77.  $e^{e^x} + C$
78.  $\ln(1 + \sin(x))$
79.  $\frac{x^2}{2} - \ln(x - 1)$
80.  $\frac{\ln(x)}{a^2} - \frac{\ln(x^2+a^2)}{2a^2}$
81.  $\lim_{t \rightarrow 1} -\frac{\sqrt{1-x^2}}{x}\Big|_{1/2}^t = \sqrt{3}$
82.  $\lim_{t \rightarrow \infty} -\frac{1+\ln(x)}{x}\Big|_1^t = 1$
83.  $-x^2 \cos x^2 + \sin x^2 + C$
84.  $\frac{1}{2}\ln\left(\frac{2-\sqrt{2}}{3(\sqrt{2}+2)}\right)$
85.  $\frac{x^2(\ln x)^2}{2} - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$
86.  $3(\ln 2 - \frac{1}{2})$
87.  $\frac{1}{3}\ln 2$
88.  $\tan x - x + C$
89.  $\frac{\pi}{12}$
90.  $\frac{\pi}{2}$
91.  $-\frac{1}{36}$
92.  $\frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{e}}\right)$
93.  $\sqrt{x^2 + 3} + C$
94.  $x^2/2 + C$
95.  $1/2\ln(x^2 + 1) - \ln|x + 1| + C$
96.  $\pi/16$
97.  $t \ln t - t + C$ , where  $t = \sin x$ .
98.  $2(t - \arctan(t)) + C$ , where  $t = \sqrt{x}$
99.  $(2/3)t^{3/2} + C$ , where  $t = \ln x$
100.  $(x^2 - 2x + 2)e^x + C$
101.  $(\cos x)^3/3 - \cos x + C$
102.  $x \ln(x^2 + 1) - 2x + 2 \arctan(x) + C$
103.  $t \ln t - t + C$ , where  $t = \ln x$