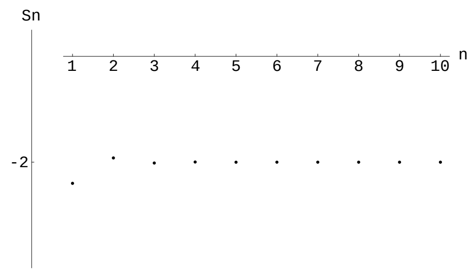
Mathematics 1b - Solution Set for PS 1
Problem Set # 1

Do: §8.2 # 3,6 (where for these two problems the instructions "Graph both the sequence of terms and the sequence of partial sums on the same screen " is changed to "Plot the partial sums.") # 12, 16,33

3) $\sum_{n=1}^{\infty} \frac{12}{(-5)^n}$

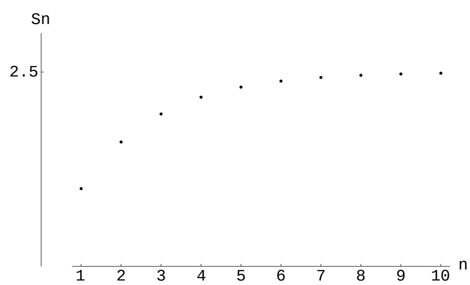
Partial Sums: -2.4, -1.92, -2.016, -1.99680, -2.00064, -1.99987, -2.00003, -1.99999, -2, -2



From the graph and the table, it seems that the series converges to -2. In fact, it is a geometric series with $a = -2.4$ and $r = -\frac{1}{5}$, so its sum is $\sum_{n=1}^{\infty} \frac{12}{(-5)^n} = \frac{-2.4}{1-(-\frac{1}{5})} = \frac{-2.4}{1.2} = -2$

6) $\sum_{n=1}^{\infty} (0.6)^{n-1}$

Partial Sums: 1.0, 1.6, 1.96, 2.176, 2.3056, 2.38336, 2.43002, 2.45801, 2.47481, 2.48488



From the graph and the table, it seems that the series converges to 2.5. In fact, it is a geometric series with $a = 1$ and $r = 0.6$, so its sum is $\sum_{n=1}^{\infty} (0.6)^{n-1} = \frac{1}{1-0.6} = \frac{1}{\frac{2}{5}} = 2.5$

12) $1 + 0.4 + 0.16 + 0.064 + ...$ is a geometric series with ratio 0.4. The series converges to $\frac{a}{1-r} = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$

16) $\sum_{n=1}^{\infty} (\frac{1}{e^2})^n \quad a = \frac{1}{e^2} = |r| < 1$, so the series converges to $\frac{\frac{1}{e^2}}{1-\frac{1}{e^2}} = \frac{1}{e^2-1}$.

33) $\sum_{n=1}^{\infty} (\frac{x}{3})^n$ is a geometric series with $r = \frac{x}{3}$, so the series converges $\quad |r| < 1 \rightarrow \frac{|x|}{3} < 1 \rightarrow |x| < 3$. Thus, $-3 < x < 3$. The sum is $\frac{\frac{x}{3}}{1-\frac{x}{3}} = \frac{x}{3-x}$.

Additional problems:

1. Determine whether or not the following sums are geometric. If the sum is geometric, express the sum in closed form.

a) $\sum_{k=1}^{70} (\frac{1}{k})$ b) $\sum_{k=1}^{50} (\frac{1}{k})^2$ c) $\sum_{k=1}^{60} (\frac{1}{k})^k$ d) $\sum_{k=1}^{60} (1.01)^{k/12}$

(a), (b), (c) are not geometric.
(d) $r = 1.01^{1/12} = 1.00083; a = 1$
$Sum = \frac{a*(1-r^{n+1})}{1-r} - 1 = \frac{1-1.01^{\frac{61}{12}}}{1-1.01^{1/12}} - 1 = 61.54.$

2. Find the sum of each of the following.

a) $\sum_{k=0}^{100}(\frac{1}{3})^k = \frac{1-(\frac{1}{3})^{101}}{1-\frac{1}{3}} = 1.5$

b) $\sum_{k=0}^{\infty}(\frac{1}{3})^k = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = 1.5$

c) $\sum_{k=2}^{100}(\frac{1}{3})^k = \frac{1-\frac{1}{3})^{101}}{1-\frac{1}{3}} - (\frac{1}{3})^0 - (\frac{1}{3})^1 = 1.5 - 1 - \frac{1}{3} = \frac{1}{6} = 0.166...$

d) $\sum_{k=2}^{\infty}(\frac{1}{3})^k = \frac{1}{1-\frac{1}{3}} - (\frac{1}{3})^0 - (\frac{1}{3})^1 = 1.5 - 1 - \frac{1}{3} = \frac{1}{6} = 0.166...$

Question: Do the first few terms of the series affect whether or not the series converges? Do the first few terms of a convergent series affect its sum?

No, the first few terms do not affect convergence because they can only finitely affect the sum. Thus, if a series converges, a finite change to the terms will still create a finite sum. Likewise, if the series diverges, a finite change will not allow the series to converge to a finite sum. Yes, the first few terms affect the sum; in fact, they typically affect the sum more greatly than terms further along because the general tendency of any converging series is to have terms that approach zero as the series continues. Therefore, the first few terms are often the greatest in magnitude. (Students do not need to use the same logic, but they must explain their answers - not just a simple "yes" or "no" - to get full credit. Of course, their logic must make sense to you.)

Think, and think again:

1. Suppose you know that the infinite series $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ converges and that $a_k > 0$ for k any positive integer. Let $s_n = a_1 + a_2 + a_3 + \cdots + a_n$. For each of the following statements, determine whether the statement must be true, could possibly be true, or must be false.

   1. $\lim_{n\to\infty} a_n = 0$ - TRUE because terms must approach zero as n approaches infinity or else the series could not summate to a finite value.
   2. $\lim_{n\to\infty} s_n = 0$ - FALSE because all terms are positive. Thus, any sum of these terms can not be zero, nor can they approach zero.
   3. There exists a number M such that $s_n < M$ for all n.

      (This is equivalent to saying that the partial sums are bounded.) - TRUE because 1) s_n is always increasing towards its infinite sum (since all terms are positive) and 2) the series converges. Thus, we know for sure that any $M > \lim_{n\to\infty} s_n$ will be greater than all s_n regardless of the value of n. (Note: If we did not know that all terms were positive, we could not be certain that a value of $M > \lim_{n\to\infty} s_n$ would be necessarily greater than a s_n value at any n.
   4. $\sum_{k=5}^{\infty} a_k$ converges - TRUE because you are merely subtracting the first four terms (a finite value) from the final sum (a finite value), creating a series that has a finite sum and thus converges.

2. Suppose you know that $\lim_{n\to\infty} b_n = 0$. Can you be sure that the infinite series

   $b_1 + b_2 + b_3 + \cdots + b_n + \cdots$ converges? NO. Although terms must approach zero or else the series would never converge, the converse is not necessarily true - if numbers approach zero slowly, they can still cause the series to diverge despite having terms that approach zero.

For the Think and Think Again questions, remember to give full credit the first time if they showed some effort in understanding the issues. Only the second time will they have points deducted if they do not understand these questions.