PROBLEM SET #13

§5.5: # 4, 5, 8, 10, 14, 16, 20, 24, 27, 28, 32

4. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ Let $u=\sqrt{x}$. Then $du=\frac{1}{2\sqrt{x}}\,dx$, so $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u\,(2\,du) =$
$2(-\cos u)+C = \boxed{-2\cos\sqrt{x}+C}$.

5. $\int \frac{4}{(1+2x)^3}\,dx$ Let $u=1+2x$. Then $du=2\,dx$, so $\int \frac{4}{(1+2x)^3}\,dx = 4\int u^{-3}\left(\frac{1}{2}du\right) =$
$2\,\frac{u^{-2}}{-2}+C = -\frac{1}{u^2}+C = \boxed{-\frac{1}{(1+2x)^2}+C}$.

8. $\int xe^{x^2}\,dx$ Let $u=x^2$. Then $du=2x\,dx$, so $\int xe^{x^2}dx = \int e^u\left(\frac{1}{2}du\right) =$
$\frac{1}{2}e^u+C = \boxed{\frac{1}{2}e^{x^2}+C}$.

10. $\int x^3(1-x^4)^5\,dx$ Let $u=1-x^4$. Then $du=-4x^3dx$, so $\int x^3(1-x^4)^5dx =$
$\int u^5\left(-\frac{1}{4}du\right) = -\frac{1}{4}\left(\frac{1}{6}u^6\right)+C = \boxed{-\frac{1}{24}(1-x^4)^6+C}$.

14. $\int \frac{x}{x^2+1}\,dx$ Let $u=x^2+1$. Then $du=2x\,dx$, so $\int \frac{x}{x^2+1}\,dx = \int \frac{\frac{1}{2}du}{u} =$
$\frac{1}{2}\ln|u|+C = \frac{1}{2}\ln|x^2+1|+C = \boxed{\frac{1}{2}\ln(x^2+1)+C}$ since $[x^2+1>0]$
or $\boxed{\ln\sqrt{x^2+1}+C}$.

16. $\int t^2\cos(1-t^3)\,dt$ Let $u=1-t^3$. Then $du=-3t^2dt$, so $\int t^2\cos(1-t^3)\,dt =$
$\int \cos u\left(-\frac{1}{3}du\right) = -\frac{1}{3}\sin u+C = \boxed{-\frac{1}{3}\sin(1-t^3)+C}$.

20. $\int \frac{\tan^{-1}x}{1+x^2}\,dx$ Let $u=\tan^{-1}x$. Then $du=\frac{dx}{1+x^2}$, so $\int \frac{\tan^{-1}x}{1+x^2}\,dx =$
$\int u\,du = \frac{u^2}{2}+C = \boxed{\frac{(\tan^{-1}x)^2}{2}+C}$.

24. $\int \frac{\cos(\pi/x)}{x^2}\,dx$ Let $u=\pi/x$. Then $du=\frac{-\pi}{x^2}\,dx$, so $\int \frac{\cos(\pi/x)}{x^2}\,dx =$
$\int \cos u\left(-\frac{1}{\pi}du\right) = -\frac{1}{\pi}\sin u+C = \boxed{-\frac{1}{\pi}\sin\frac{\pi}{x}+C}$.

27. $\displaystyle\int \frac{e^x+1}{e^x}\,dx = \int (1+e^{-x})\,dx = \boxed{x - e^{-x} + C}$ [Substitute $u = -x$].

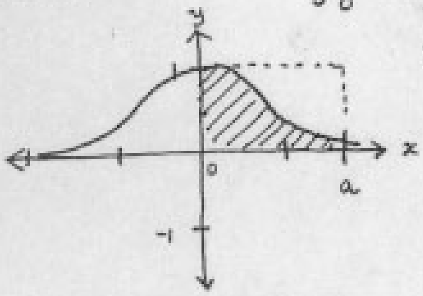
28. $\displaystyle\int \frac{e^x}{e^x+1}\,dx$ Let $u = e^x+1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} =$

$\ln|u| + C = \boxed{\ln(e^x+1)+C}$.

32. $\displaystyle\int \frac{x}{1+x^4}\,dx$ Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{1+x^4}\,dx = \int \frac{\frac{1}{2}\,du}{1+u^2} =$

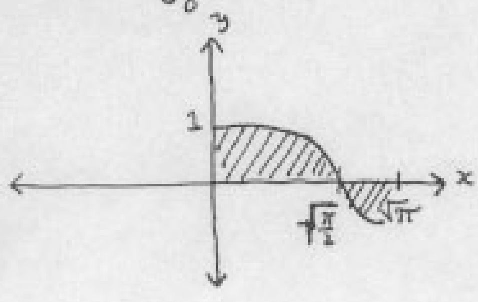
$\frac{1}{2}\tan^{-1} u + C = \boxed{\frac{1}{2}\tan^{-1}(x^2)+C}$.

• Claim 1 $\displaystyle 0 < \int_0^a e^{-x^2}\,dx < a$



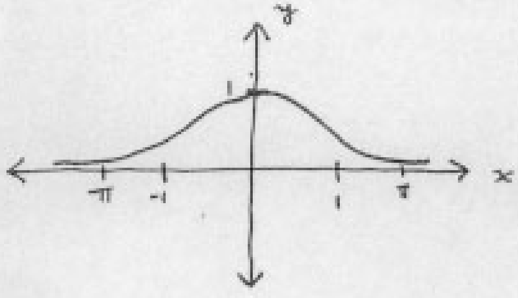
The rectangle has an area of a, since its base has a length of a and it has a height of 1. Clearly, the rectangle has a greater area than $\int_0^a e^{-x^2}dx$, so the claim is $\boxed{\text{TRUE}}$.

• Claim 2 $\displaystyle\int_0^{\sqrt{\pi}} \cos(x^2)\,dx < 0$



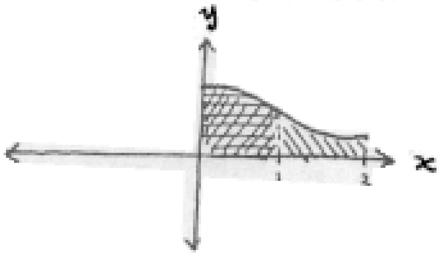
The part of the integral from $0 \to \sqrt{\pi/2}$ is positive, and larger in area than the integral from $\sqrt{\pi/2} \to \sqrt{\pi}$. So, the net result is clearly positive. Therefore, the claim is $\boxed{\text{FALSE}}$.

• Claim 3 $\displaystyle\int_{-\pi}^{\pi} e^{-x^2/\sqrt{2}}\,dx = 2\int_0^{\pi} e^{-x^2/\sqrt{2}}\,dx$



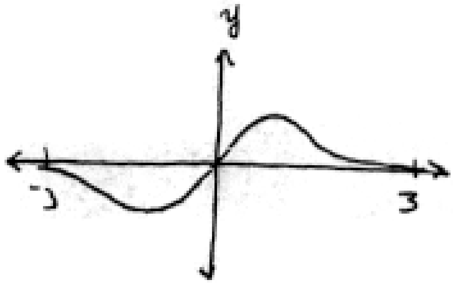
Because the integration is symmetric about the y-axis, and both $\int_{-\pi}^{0}$ and $\int_0^{\pi}$ are positive, we do indeed have that $\int_{-\pi}^{\pi} = 2\int_0^{\pi}$. The claim is $\boxed{\text{TRUE}}$.

• Claim 4: $\int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx < \int_1^2 \frac{1}{\sqrt{1+x^4}}\,dx$



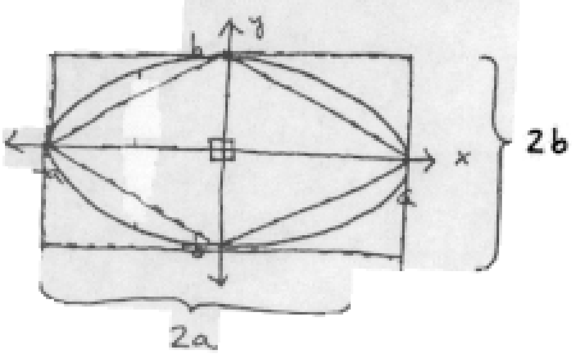
Clearly, the area under the curve from $0 \to 1$ is greater than the area under the curve from $1-2$. So, the claim is [redacted] $\boxed{\text{FALSE}}$.

Claim 5: $\int_{-3}^{3} \frac{x}{+x^4}\,dx > 0.001$



is symmetric, and one ½ of the integral • measures an area above the x-axis and the other ½ of the integral measures an area below the x-axis $\left(\int_{-3}^{0}\right)$, we have that the two halves cancel out, and the integral is 0. Since 0.001, the claim is $\boxed{\text{FALSE}}$.

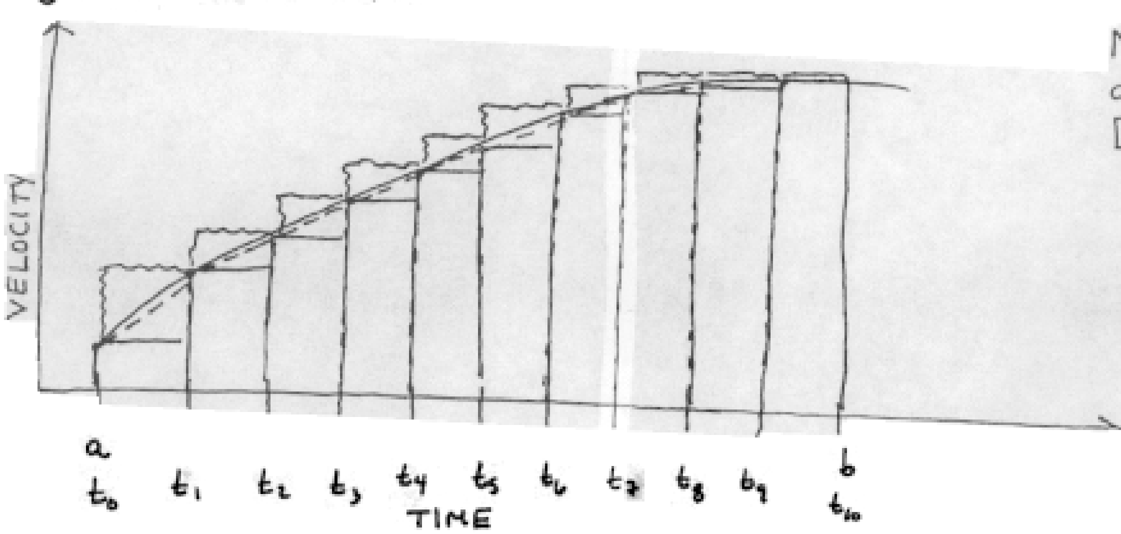
• Claim 6 The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$ is greater than 2ab and less than 4ab



Clearly the rectangle of area 4ab is larger than the ellipse. Moreover, the area of the right triangles, collectively, is $4 \times \frac{1}{2}ab = 2ab$. Therefore, the area of the ellipse is A, with $2ab < A < 4ab$.

The claim is $\boxed{\text{true}}$.

(Diagram for Claims 7–8).



Note: the distance travelled on $[a,b] = \int_a^b v(t)$.

[redacted]

• Claim 7: $\sum_{k=1}^{10} v(t_{k-1})\,\Delta t$ > the distance travelled on $[a, b]$

$$\sum_{k=1}^{10} v(t_{k-1})\Delta t = \left[v(t_0) + v(t_1) + v(t_2) + \dots v(t_9)\right]\Delta t$$

This represents the rectangles inscribed under the curve, which is clearly less than $\int_a^b v(t)$ So, the claim is $\boxed{\text{FALSE}}$.

• Claim 8: $\sum_{k=1}^{10} v(t_k)\,\Delta t$ > the distance travelled on $[a, b]$

$$\sum_{k=1}^{10} v(t_k)\Delta t = \left[v(t_1) + v(t_2) + v(t_3) + \dots + v(_{10})\right]\Delta t$$

This represents the rectangles circumscribed above the curve, which is clearly geater than $\int_a^b v(t)$. So, the claim is $\boxed{\text{TRUE}}$.

laim 9: $\frac{1}{2}\left[\sum_{k=1}^{10} v(t_k \;\Delta t + \sum_{k=1}^{10} v(t_{k-1})\Delta t\right]$

This is $\frac{1}{2}\left[v(t_1) + v(t_2) + v(t_3) + \dots + v(t_{10}) + v(t_0) + v(t_1) + \dots + v(t_9)\right]\Delta t$

which is $\frac{1}{2}\left[v(t_0) + 2v(t_1) + \dots + 2v(t_9) + v(t_{10})\right]\Delta t$ This is the trapezoidal rule! Looking at the graph, we see that this sum is less than the integral from $a \to b$ (i.e., the distance travelled

So, the claim is $\boxed{\text{TRUE}}$.