PROBLEM SET #14

§ 5.5 : # [redacted], 60

§ 5.6 : # 4, 5, 8, 14, 22, 28

§ Chapter 5 Review : # 5

§ 5.6 : # 34 (EXTRA-CREDIT)

60. Number of calculators : $x(4) - x(2) = \int_2^4 5000\left[1 - 100(t+10)^{-2}\right] dt =$

$5000\left[t + 100(t-10)^{-1}\right]_2^4 = 5000\left[\left(4 + \frac{100}{14}\right) - \left(2 + \frac{100}{12}\right)\right] \approx \boxed{4048}$

4. $\int x^4 \ln x \, dx$ Let $u = \ln x$, $dv = x^4 dx \Rightarrow du = (1/x)\,dx$, $v = \frac{1}{5}x^5$. Then

$\int x^4 \ln x \, dx = 1/5\, x^5 \ln x - \int 1/5\, x^5 (1/x)\, dx =$ [redacted] $1/5\, x^5 \ln x - 1/5 \int x^4 dx =$

$1/5\, x^5 \ln x - 1/5\,(1/5\, x^5) + C = \boxed{1/5\, x^5 \ln x - \frac{1}{25}x^5 + C}$ or $\boxed{\frac{1}{25}x^5(5\ln x - 1) + C}$.

5. $\int x \sin 4x \, dx$ Let $u = x$, $dv = \sin 4x\, dx \Rightarrow du = dx$, $v = -\frac{1}{4}\cos 4x$ Then

by Equation 2, $\int x \sin 4x \, dx = -\frac{1}{4} x \cos 4x - \int\left(-\frac{1}{4}\cos 4x\right) dx = -\frac{1}{4} x \cos 4x +$

$\frac{1}{4}\left(\frac{1}{4}\sin 4x\right) + C = \boxed{-\frac{1}{4} x \cos 4x + \frac{1}{16}\sin 4x + C.}$

8. $\int x^2 \sin ax\, dx$ First let $u = x^2$, $dv = \sin ax\, dx \Rightarrow du = 2x\, dx$, $v = -\frac{1}{a}\cos ax$.

Then, by Equation 2, $I = \int x^2 \sin ax\, dx = -\frac{x^2}{a}\cos ax - \int \left(-\frac{1}{a}\right)\cos ax\,(2x\, dx) =$

$-\frac{x^2}{a}\cos(ax) + \frac{2}{a}\int x\cos ax\, dx$. Next let $U = x$, $dV = \cos ax\, dx \Rightarrow$

$dU = dx$, $V = \frac{1}{a}\sin ax$. So $\int x\cos ax\, dx = \frac{x}{a}\sin ax - \int \frac{1}{a}\sin ax\, dx =$

$\frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax + C_1$. Substituting for $\int x\cos ax\, dx$, we get

$I = -\frac{x^2}{a}\cos ax + \frac{2}{a}\left(\frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax + C_1\right) =$

$\boxed{-\frac{x^2}{a}\cos ax + \frac{2x}{a^2}\sin ax + \frac{2}{a^3}\cos ax + C}$.

14. $\int e^{-\theta}\cos 2\theta\, d\theta$ First let $u = e^{-\theta}$, $dv = \cos 2\theta\, d\theta \Rightarrow du = -e^{-\theta}d\theta$, $v =$
$\frac{1}{2}\sin 2\theta$. Then $I = \int e^{-\theta}\cos 2\theta\, d\theta = \frac{1}{2}e^{-\theta}\sin 2\theta - \int \frac{1}{2}\sin 2\theta\,(-e^{-\theta}d\theta) =$
$\frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\int e^{-\theta}\sin 2\theta\, d\theta$. Next let $U = e^{-\theta}$, $dV = \sin 2\theta\, d\theta \Rightarrow$
$dU = -e^{-\theta}d\theta$, $V = -\frac{1}{2}\cos 2\theta$, so $\int e^{-\theta}\sin 2\theta\, d\theta = -\frac{1}{2}e^{-\theta}\cos 2\theta - \int\left(-\frac{1}{2}\right)\cos 2\theta(-e^{-\theta}d\theta)$
$= -\frac{1}{2}e^{-\theta}\cos 2\theta - \frac{1}{2}\int e^{-\theta}\cos 2\theta\, d\theta$. So $I = \frac{1}{2}e^{-\theta}\sin 2\theta + \frac{1}{2}\left[\left(-\frac{1}{2}e^{-\theta}\cos 2\theta\right) - \frac{1}{2}I\right] =$
$\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta - \frac{1}{4}I \Rightarrow \frac{5}{4}I = \frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1 \Rightarrow$
$I = \frac{4}{5}\left(\frac{1}{2}e^{-\theta}\sin 2\theta - \frac{1}{4}e^{-\theta}\cos 2\theta + C_1\right) = \boxed{\frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C}$.

22. $\int x\tan^{-1}x\, dx$ Let $u = \tan^{-1}x$, $dv = x\, dx \Rightarrow du = dx/(1+x^2)$, $v = \frac{1}{2}x^2$.
Then $\int x\tan^{-1}x\, dx = \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$. $\int \frac{x^2}{1+x^2}dx = \int \frac{(1+x^2)-1}{(1+x^2)}dx =$
$\int 1\, dx - \int \frac{1}{1+x^2}dx = x - \tan^{-1}x + C_1$. So $\int x\tan^{-1}x\, dx = \frac{1}{2}x^2\tan^{-1}x -$
$\frac{1}{2}(x - \tan^{-1}x + C_1) = \boxed{\frac{1}{2}(x^2\tan^{-1}x + \tan^{-1}x - x) + C}$.

28. $\int_1^4 e^{\sqrt{x}}\,dx$ Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w\,dw$. Thus, $\int_1^4 e^{\sqrt{x}}\,dx =$ $\int_1^2 e^w\, 2w\,dw$. Now use parts with $u = 2w$, $dv = e^w dw$, $du = 2dw$, $v = e^w$ to get $\int_1^2 e^w\, 2w\,dw = \left[2we^w\right]_1^2 - 2\int_1^2 e^w\,dw = 4e^2 - 2e - 2(e^2 - e) =$

$\boxed{2e^2}$

5. $\boxed{\text{FALSE}}$ For example, let $f(x) = x^2$. Then $\int_0^1 \sqrt{x^2}\,dx = \int_0^1 x\,dx = \frac{1}{2}$, but $\sqrt{\int_0^1 x^2\,dx} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$.

34. (a) Let $u = \cos^{n-1} x$, $dv = \cos x\,dx \Rightarrow du = -(n-1)\cos^{n-2}x \sin x\,dx$, $v = \sin x$ in (2):

$$\begin{aligned}\int \cos^n x\,dx &= \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x \sin^2 x\,dx \\ &= \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x(1-\cos^2 x)\,dx \\ &= \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\int \cos^n x\,dx\end{aligned}$$

Rearranging terms gives $n\int \cos^n x\,dx = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx$ or

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

(b) Take $n=2$ in part (a) to get $\int \cos^2 x\,dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}\int 1\,dx =$

$\boxed{\frac{x}{2} + \frac{\sin 2x}{4} + C}$.

(c) $\int \cos^4 x\,dx = \frac{1}{4}\cos^3 x\sin x + \frac{3}{4}\int \cos^2 x\,dx = \boxed{\frac{1}{4}\cos^3 x \sin x + \frac{3}{8}x + \frac{3}{16}\sin 2x + C}$