6.3

1. $y = 2 - 3x \;\Rightarrow\; L = \int_{-2}^{1} \sqrt{1 + (dy/dx)^2}\,dx = \int_{-2}^{1} \sqrt{1 + (-3)^2}\,dx = \sqrt{10}\,[1 - (-2)] = 3\sqrt{10}.$

The arc length can be calculated using the distance formula, since the curve is a line segment, so

$$L = [\text{distance from } (-2, 8) \text{ to } (1, -1)] = \sqrt{[1 - (-2)]^2 + [(-1) - 8]^2} = \sqrt{90} = 3\sqrt{10}$$

4. $y = 2^x \;\Rightarrow\; dy/dx = (2^x)\ln 2 \;\Rightarrow\; L = \int_0^3 \sqrt{1 + (\ln 2)^2\, 2^{2x}}\,dx$

21. The prey hits the ground when $y = 0 \;\Leftrightarrow\; 180 - \frac{1}{45}x^2 = 0 \;\Leftrightarrow\; x^2 = 45 \cdot 180 \;\Rightarrow\; x = \sqrt{8100} = 90$, since x must be positive. $y' = -\frac{2}{45}x \;\Rightarrow\; 1 + (y')^2 = 1 + \frac{4}{45^2}x^2$, so the distance traveled by the prey is

$$\begin{aligned} L &= \int_0^{90} \sqrt{1 + \tfrac{4}{45^2}x^2}\,dx = \int_0^4 \sqrt{1 + u^2}\left(\tfrac{45}{2}\,du\right) \quad \left[u = \tfrac{2}{45}x,\ du = \tfrac{2}{45}\,dx\right] \\ &\overset{21}{=} \tfrac{45}{2}\left[\tfrac{1}{2}u\sqrt{1 + u^2} + \tfrac{1}{2}\ln\!\left(u + \sqrt{1 + u^2}\right)\right]_0^4 \\ &= \tfrac{45}{2}\left[2\sqrt{17} + \tfrac{1}{2}\ln\!\left(4 + \sqrt{17}\right)\right] = 45\sqrt{17} + \tfrac{45}{4}\ln\!\left(4 + \sqrt{17}\right) \approx 209.1 \text{ m} \end{aligned}$$

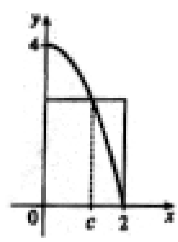
6.4

2. $g_{ave} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{2}{9}\left[x^{3/2}\right]_1^4 = \frac{2}{9}(8 - 1) = \frac{14}{9}$

5. (a) $$\begin{aligned} f_{ave} &= \tfrac{1}{2-0}\int_0^2 (4 - x^2)\,dx \\ &= \tfrac{1}{2}\left[4x - \tfrac{1}{3}x^3\right]_0^2 \\ &= \tfrac{1}{2}\left(8 - \tfrac{8}{3}\right) = \tfrac{8}{3} \end{aligned}$$

(b) $f_{ave} = f(c) \;\Leftrightarrow\; \frac{8}{3} = 4 - c^2 \;\Leftrightarrow\; c^2 = \frac{4}{3}$
$\;\Leftrightarrow\; c = \frac{2}{\sqrt{3}} \approx 1.15$

(c)



11. Let $t = 0$ and $t = 12$ correspond to 9 A.M. and 9 P.M., respectively.

$$\begin{aligned} T_{ave} &= \tfrac{1}{12-0}\int_0^{12}\left[50 + 14\sin\tfrac{1}{12}\pi t\right]dt = \tfrac{1}{12}\left[50t - 14\cdot\tfrac{12}{\pi}\cos\tfrac{1}{12}\pi t\right]_0^{12} \\ &= \tfrac{1}{12}\left[50\cdot 12 + 14\cdot\tfrac{12}{\pi} + 14\cdot\tfrac{12}{\pi}\right] = \left(50 + \tfrac{28}{\pi}\right)\,^\circ\text{F} \approx 59\,^\circ\text{F} \end{aligned}$$

16. $$v_{ave} = \frac{1}{R-0}\int_0^R v(r)\,dr = \frac{1}{R}\int_0^R \frac{P}{4\eta l}(R^2 - r^2)\,dr = \frac{P}{4\eta lR}\left[R^2 r - \tfrac{1}{3}r^3\right]_0^R = \frac{P}{4\eta lR}\left(\tfrac{2}{3}\right)R^3 = \frac{PR^2}{6\eta l}.$$

Since $v(r)$ is decreasing on $(0, R]$, $v_{max} = v(0) = \dfrac{PR^2}{4\eta l}$. Thus, $v_{ave} = \frac{2}{3}v_{max}$.

6.5

$$= 10\left[-\frac{1}{u}\right]_1^{10} = 10\left(-\tfrac{1}{10} + 1\right) = 9 \text{ ft-lb}$$