Solution for Handout K, Problem 1

1. A canister contains 10 liters of blue paint. Paint is being used at a rate of 2 liters per hour and the canister is being replenished by pale blue paint that is 80% blue paint and 20% percent white paint. Assume the canister is well-mixed and that paint is both entering and leaving the canister continuously.

   a) Write a differential equation whose solution is $w(t)$, the amount of white paint in the canister at time t. Specify the initial condition. Use qualitative analysis (and common sense) to sketch the solution to the differential equation.
   $\frac{dw}{dt} = 0.4L/hr - 0.2 * w(t)$ (Unit label is not necessary.)
   $w(0) = 0L$
   A description of what the sketch should look like: There is a nullcline at $w = 2$ that is stable. Because the initial condition is $w(0) = 0$, the graph should begin at $(0,0)$ and increase as a logistic curve, plateauing as it nears the nullcline at $w = 2$.

   b) Write a differential equation whose solution is $b(t)$, the amount of blue paint in the canister at time t. Specify the initial condition. Use qualitative analysis (and common sense) to sketch the solution to the differential equation.
   $\frac{db}{dt} = 1.6L/hr - 0.2 * b(t)$ (Unit label is not necessary.)
   $b(0) = 10L$
   A description of what the sketch should look like: There is a nullcline at $b = 8$ that is stable. Because the initial condition is $b(0) = 10$, the graph should begin at $(0,10)$ and decrease as a logistic curve, flattening out as it nears the nullcline at $b = 8$.