52. $\int \frac{3}{x^2(x^2+9)}\,dx = 3\int\left(\frac{A}{x^2}+\frac{B}{x^2+9}\right)dx = \int \frac{\frac{1}{3}}{x^2}\,dx - \int \frac{\frac{1}{3}}{x^2+9}\,dx$

$x^2=0: \; x^2=-9$

$A=\frac{1}{9}: \; B=-\frac{1}{9}$

$= \frac{1}{3}\left(-\frac{1}{x}\right) - \frac{1}{3}\arctan\left(\frac{x}{3}\right)\cdot\frac{1}{9} + C$

$= \boxed{-\frac{1}{3x} - \frac{\arctan\left(\frac{x}{3}\right)}{27} + C}$

53. $\int \frac{x^3}{x^2+1}\,dx = \frac{x^2}{2}\cdot\ln|x^2+1| - \int x\cdot\ln|x^2+1|\,dx = \frac{x^2}{2}\cdot\ln|x^2+1|$

$u=x^2 \quad dv=\frac{x}{x^2+1}\,dx$

$du=2x\,dx \quad v=\frac{1}{2}\ln|x^2+1|$

$-\frac{1}{2}\cdot(x^2+1)\ln|x^2+1| + \frac{x^2}{2} + \frac{1}{2} + C_1$

$= \boxed{\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + C}$

54. $\int \frac{dx}{x^3+x} = \int\left(\frac{A}{x}+\frac{B}{x^2+1}\right)dx = \int\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx =$

$x=0: \quad x^2=-1$

$A=1 \quad Bx=1=-x^2$

$B=-x$

$= \boxed{\ln|x| - \frac{1}{2}\ln|x^2+1| + C}$

55. $\int \frac{\sin x\,dx}{\cos^2(x)-5\cos(x)+4} = \int \frac{-du}{u^2-5u+4} = \int\left(\frac{-1}{(u-4)(u-1)}\right)du = \int\left(\frac{A}{u-4}+\frac{B}{u-1}\right)du$

$u=\cos(x)$, $du=-\sin(x)\,dx$

$A=-\frac{1}{3}$, $B=+\frac{1}{3}$ $\quad = \int\left(\frac{-\frac{1}{3}}{u-4}+\frac{\frac{1}{3}}{u-1}\right)du = \frac{1}{3}\ln\left|\frac{u-1}{u-4}\right| + C$

$= \boxed{\frac{1}{3}\ln\left|\frac{\cos(x)-1}{\cos(x)-4}\right| + C}$

56. $\int \frac{dx}{e^x-1} = \int \frac{e^x-e^x+1}{e^x-1}\,dx = \int \frac{e^x}{e^x-1}\,dx - \int \frac{e^x-1}{e^x-1}\,dx$

$= \boxed{\ln|e^x-1| - x + C}$

57. $\int \frac{x^4-2x^3+4x^2-4}{x^2-x-2}dx = \int\left[\frac{x^3(x-2)}{(x-2)(x+1)} + 4\cdot\frac{x^2-1}{(x-2)(x+1)}\right]dx$

$= \int\left(\frac{x^3}{x+1} + \frac{4(x-1)}{x-2}\right)dx$

$(x^3+1)\div(x+1)$:

$$\begin{array}{rrrr|r} 1 & 0 & 0 & 1 & -1 \\ & -1 & 1 & -1 & \\ \hline 1 & -1 & 1 & 0 & \end{array}$$

$= x^2-x+1$

$= \int\left[\frac{x^3+1}{x+1} - \frac{1}{x+1} + \frac{4(x-2)}{x-2} + \frac{4}{x-2}\right]dx$

$= \int\left[x^2-x+1-\frac{1}{x+1}+4+\frac{4}{x-2}\right]dx$

$= \boxed{\frac{x^3}{3}-\frac{x^2}{2}+5x+4\ln|x-2|-\ln|x+1|+C}$

58. $\int_0^1 \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1 = \frac{\pi}{6}-0 = \boxed{\frac{\pi}{6}}$

59. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\cdot e^{\sqrt{x}}dx = \boxed{2e^{\sqrt{x}}+C}$

60. $\int \sqrt{9-4x^2}\,dx = \frac{1}{2}\left(x\sqrt{9-4x^2}+\frac{9}{2}\sin^{-1}\left(\frac{2x}{3}\right)\right)+C$

$a=3,\ u=2x,\ du=2dx$

$= \boxed{\frac{x}{2}\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x}{3}\right)+C}$

61. $\int \frac{x+1}{x^2+1}dx = \int\frac{x}{x^2+1}dx + \int\frac{dx}{x^2+1} = \boxed{\frac{1}{2}\ln|x^2+1|+\arctan(x)+C}$

62. $\int\left(2xe^{(x+1)^2}+2e^{(x+1)^2}\right)dx = \int(2x+2)\left(e^{(x^2+2x+1)}\right)dx$

$= \boxed{e^{(x+1)^2}+C}$

63. $\int(\cos^2(x)-\sin^2(x))dx = \int\cos(2x)dx = \boxed{\frac{\sin(2x)}{2}+C}$

64. $\int \frac{x}{\sqrt{1-x^2}}\,dx = \int x(1-x^2)^{-\frac{1}{2}}\,dx = \boxed{-(1-x^2)^{\frac{1}{2}}+C}$

65. $\int x\sin((x+1)(x-1))\,dx = \int x\sin(x^2-1)\,dx = \boxed{-\frac{1}{2}\cos(x^2-1)+C}$

66. $\int \frac{x^5-1}{x-1}\,dx = \int (x^4+x^3+x^2+x+1)\,dx$

$(x^5-1)\div(x-1)$:

$= \boxed{\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x+C}$

$$\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ & 1 & 1 & 1 & 1 & 1 & \\ \hline 1 & 1 & 1 & 1 & 1 & 0 & \end{array}$$

$= x^4+x^3+x^2+x+1$

67. $\int 2x((x^2+1)^2+1)^2\,dx = \int (u^2+1)^2\,du = \int (u^4+2u^2+1)\,du$

$u=(x^2+1)$, $du=2x\,dx$

$= \frac{u^5}{5}+\frac{2u^3}{3}+u+C = \boxed{\frac{(x^2+1)^5}{5}+\frac{2(x^2+1)}{3}+x^2+C}$

68. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^3}{\sqrt{1-x^2}}\,dx = -x^2(1-x^2)^{\frac{1}{2}}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x(1-x^2)^{\frac{1}{2}}\,dx$

$u=x^2$ $\quad dv=\frac{x}{\sqrt{1-x^2}}\,dx$

$du=2x\,dx$ $\quad v=-(1-x^2)^{\frac{1}{2}}$

$= \left[-x^2\sqrt{1-x^2}-2(1-x^2)^{3/2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

No odd-number power of variable, so final sum $= \boxed{0}$ (since $(\frac{\pi}{2})^2 = (-\frac{\pi}{2})^2$).

69. $\int_1^e x^2\ln x\,dx = \frac{x^3}{3^2}\left[3\ln x-1\right]\Big|_1^e = \frac{e^3}{9}[3-1]-\frac{1}{9}[-1] = \boxed{\frac{2e^3+1}{9}}$

70. $\int \sin x\cos x\,dx = \boxed{\frac{\sin^2 x}{2}+C}$ ($\sin x = u$, $\cos x = du$)

71. $\int \frac{dx}{x\ln x\cdot\ln(\ln x)} = \ln|\ln|\ln x|| + C$ → Think backwards using derivation ⇒ makes it easier.

72. $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}} = \frac{x}{\sqrt{1-x^2}} + C$

$a=1,\ u=x$

73. $\displaystyle\int \sin^3(2x)\,dx = -\frac{\sin^2(2x)\cos(2x)}{2} - \int -2\sin(2x)\cos^2(2x)\,dx$

$u = \sin^2(2x) \qquad dv = \sin(2x)$

$du = 4\sin(2x)\cos(2x)dx \qquad v = \frac{-1}{2}\cos(2x)$

$\displaystyle = -\frac{\sin^2(2x)\cos(2x)}{2} - \frac{\cos^3(2x)}{3} + C = -\frac{(1-\cos^2(2x))\cos(2x)}{2} - \frac{\cos^3(2x)}{3} + C$

$\displaystyle = \boxed{-\frac{\cos(2x)}{2} + \frac{\cos^3(2x)}{6} + C}$

74. $\displaystyle\int \ln\sqrt{x}\,dx = \int \ln u\ 2u\,du = 2\int u\ln u\,du$

$\sqrt{x} = u \qquad = 2\cdot\frac{u^2}{2^2}\left[2\ln u - 1\right] = \frac{x}{2}\left[2\ln\sqrt{x} - 1\right]$

$x = u^2$

$dx = 2u\,du \qquad = \boxed{x\ln\sqrt{x} - \frac{x}{2} + C}$

75. $\displaystyle\int \frac{x+1}{x^2-4}\,dx = \int \frac{x}{x^2-4}\,dx + \int \frac{1}{x^2-4}\,dx$

$= \frac{1}{2}\ln|x^2-4| + \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$

$= \frac{1}{2}\ln|x-2| + \frac{1}{2}\ln|x+2| + \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$

$= \boxed{\frac{3}{4}\ln|x-2| + \frac{1}{4}\ln|x+2| + C}$

76. $\displaystyle\int \frac{1}{(x^2+1)^{3/2}}\,dx = \boxed{\frac{x}{\sqrt{1+x^2}} + C}$

$a=1,\ u=x$

77. $\int e^{e^x + x}\,dx = \int e^{e^x} \cdot e^x\,dx = \boxed{e^{e^x} + C}$

78. $\int \frac{\cos(x)}{1+\sin(x)}\,dx = \int \frac{\cos(x)}{1+\sin(x)} \cdot \frac{1-\sin(x)}{1-\sin(x)}\,dx = \int \frac{\cos(x) - \sin(x)\cos(x)}{1-\sin^2(x)}\,dx$

$$= \int \frac{\cos(x) - \sin(x)\cos(x)}{\cos^2(x)}\,dx = \int \frac{1-\sin(x)}{\cos(x)}\,dx$$

$$= \int \sec x\,dx - \int \tan x\,dx = \ln|\sec x + \tan x|$$

$$- \ln|\sec x| + C = \ln\left|\frac{\sec x + \tan x}{\sec x}\right| + C$$

$$= \boxed{\ln|1+\sin x| + C}$$

79. $\int \frac{x^3+1}{x^2-1}\,dx = \int \frac{x^2-x+1}{x-1}\,dx = \int \frac{x(x-1)}{(x-1)}\,dx + \int \frac{1}{x-1}\,dx$

$(x^3+1) \div (x+1)$:

1	0	0	1	$\underline{-1}$
	-1	1	-1	
1	-1	1	0	

$= x^2 - x + 1$

$$= \boxed{\frac{x^2}{2} + \ln|x-1| + C}$$

80. $\int \frac{1}{x(x^2+a^2)}\,dx = \int \left(\frac{A}{x} + \frac{B}{x^2+a^2}\right)dx = \int \left(\frac{\frac{1}{a^2}}{x} + \frac{-x\left(\frac{1}{a^2}\right)}{x^2+a^2}\right)dx$

$A(x^2+a^2) + Bx = 1$

$x=0$: $x^2 = -a^2$

$A = \frac{1}{a^2}$ $\quad Bx = 1 = -\frac{x^2}{a^2}$

$B = -\frac{x}{a^2}$

$$= \boxed{\frac{1}{a^2}\left(\ln|x| - \frac{\ln(x^2+a^2)}{2}\right) + C}$$

81. $\int_{1/2}^{1} \frac{1}{x^2\sqrt{1-x^2}}\,dx = \lim_{a\to 1} -\frac{1}{1^2\cdot x}\sqrt{1-x^2}\Big|_{1/2}^{a} = \lim_{a\to 1} -\frac{1}{a}\sqrt{1-a^2}$

$-\left(-2\sqrt{\tfrac{3}{4}}\right) = 0 + \sqrt{3} = \boxed{\sqrt{3}}$

82. $\int_{1}^{\infty} \frac{\ln(x)}{x^2}\,dx = \frac{1}{x}\left[-\ln x - 1\right]_1^{\infty} = \lim_{a\to\infty} \frac{1}{a}(-\ln a - 1)$

$-(-1) = 0 + 1 = \boxed{1}$

$u = x,\ n = -2$

83. $\int \frac{x}{(x^2-a^2)^{3/2}}\,dx = \frac{1}{2}\int 2x(x^2-a^2)^{-3/2}\,dx = \boxed{-(x^2-a^2)^{-\frac{1}{2}} + C}$

84. $\int x^3 \sin(x^2)\,dx = -\frac{x^2}{2}\cos(x^2) + \int x\cos(x^2)\,dx$

$u = x^2 \quad dv = x\sin(x^2)$

$du = 2x\,dx \quad v = -\frac{1}{2}\cos(x^2)$

$= \boxed{-\frac{x^2}{2}\cos(x^2) + \frac{\sin(x^2)}{2} + C}$

85. $\int x(\ln x)^2\,dx = \frac{x^2}{2}(\ln x)^2 - \int \frac{x^2}{2}\cdot 2\ln x \cdot \frac{1}{x}\,dx$

$u = (\ln x)^2 \quad dv = x\,dx$

$du = 2\ln x \cdot \frac{1}{x}\,dx \quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}(\ln x)^2 - \int x \ln x\,dx$

$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{4}\left[2\ln x - 1\right] + C$

$= \boxed{\frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C}$