86. $\int_0^1 \frac{1}{1+x^{1/3}}\,dx$ let $z = 1 + x^{\frac{1}{3}}$ $dz = \frac{1}{3}x^{-\frac{2}{3}}\,dx \Rightarrow dx = 3x^{\frac{2}{3}}\,dz$

$$\int \frac{1}{1+x^{1/3}}\,dx = \int \frac{1}{z}\cdot 3x^{\frac{2}{3}}\,dz = \int \frac{1}{z}\cdot 3(z-1)^2\,dz$$

$$= 3\int \frac{z^2-2z+1}{z}\,dz = 3\left[\int z\,dz + \int -2\,dz + \int \frac{1}{z}\,dz\right]$$

$$= 3\left(\frac{z^2}{2} - 2z + \ln|z| + C\right) = \frac{3}{2}(1+x^{\frac{1}{3}})^2 - 6(1+x^{\frac{1}{3}}) + 3\ln|1+x^{\frac{1}{3}}| + C$$

$$\therefore \int_0^1 \frac{1}{1+x^{1/3}}\,dx = \left[\frac{3}{2}(1+x^{\frac{1}{3}})^2 - 6(1+x^{\frac{1}{3}}) + 3\ln|1+x^{\frac{1}{3}}|\right]_0^1$$

$$= \left[\left(\frac{3}{2}\cdot 4 - 6\cdot 2 + 3\ln 2\right) - \left(\frac{3}{2} - 6 + 0\right)\right]$$

$$= \boxed{-\frac{3}{2} + 3\ln 2}$$

87. $$\int_1^3 \frac{1}{x(x+3)}\,dx = \int_1^3 \left(\frac{A}{x} + \frac{B}{x+3}\right)dx = \int_1^3 \left(\frac{\frac{1}{3}}{x} - \frac{\frac{1}{3}}{x+3}\right)dx$$

$A(x+3) + B(x) = 1$

$x=-3$: $x=0$

$B = -\frac{1}{3}$ $A = \frac{1}{3}$

$$= \left[\frac{1}{3}\ln|x| - \frac{1}{3}\ln|x+3|\right]_1^3 = \frac{1}{3}\left(\ln\frac{x}{x+3}\right)\Big|_1^3$$

$$= \frac{1}{3}\ln\left(\frac{1}{2}\right) - \frac{1}{3}\ln\left(\frac{1}{4}\right) = \boxed{\frac{1}{3}\ln(2)}$$

88. $\int \tan^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int (\frac{1}{\cos^2 x} - 1) dx = \boxed{-x + \tan x + c}$

(note $\int \frac{1}{\cos^2 x} dx = \tan x + c$)

89. $\int_0^{\frac{1}{2}\ln 3} \frac{1}{e^x + e^{-x}} dx$ let $x = \ln z$ $\therefore e^x = z$, $e^{-x} = e^{-\ln z} = \frac{1}{z}$

$dx = \frac{1}{z} dz$

$\int \frac{1}{e^x + e^{-x}} dx = \int \frac{1}{z + \frac{1}{z}} \cdot \frac{1}{z} dz = \int \frac{1}{z^2+1} dz = \tan^{-1}(z) + c$

$= \tan^{-1}(e^x) + c$

$\therefore \int_0^{\frac{1}{2}\ln 3} \frac{1}{e^x + e^{-x}} dx = \left[\tan^{-1}(e^x) + c\right]_0^{\frac{1}{2}\ln 3} = \tan^{-1}(e^{\frac{1}{2}\ln 3}) - \tan^{-1}(e^0)$

$= \boxed{\tan^{-1}(\sqrt{3}) - \tan^{-1}(1)}$

90. $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ let $x = \sin z$ $dx = \cos z \, dz$

$\int \frac{1}{\sqrt{1-\sin^2 z}} dz = \int \frac{1}{\cos z} \cdot \cos z \, dz = \int 1 \, dz = z + c$

$= \sin^{-1}(x) + c$

$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \left[\sin^{-1}(x) + c\right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \boxed{\frac{\pi}{2}}$

91. $\int_0^1 x^5 \ln x \, dx$ let $u = \ln x$ $dv = x^5 dx$

$\int x^5 \ln x \, dx$ $du = \frac{1}{x}$ $v = \frac{x^6}{6}$

$= \frac{x^6}{6} \ln x - \int \frac{x^6}{6} \cdot \frac{1}{x} dx$

$= \frac{x^6}{6} \ln x - \frac{1}{6} \int x^5 dx = \frac{x^6}{6} \ln x - \frac{1}{36} x^6 + c$

$\int_0^1 x^5 \ln x \, dx = \lim_{b \to 0} \left[\frac{x^6}{6} \ln x - \frac{1}{36} x^6 + c\right]_b^1$

$= \lim_{b \to 0} \left[(0 - \frac{1}{36}) - (\frac{b^6}{6} \ln(b) - \frac{b^6}{36})\right] = \boxed{-\frac{1}{36}}$

92. $\int_1^2 \frac{e^{-\frac{1}{x}}}{x^2} dx$ let $u = -\frac{1}{x}$ $du = \frac{1}{x^2} dx$ $\int \frac{e^{-\frac{1}{x}}}{x^2} dx = \int \frac{e^u}{x^2} \cdot x^2 du = \int e^u du$

$= e^{-\frac{1}{x}} + c$

$\therefore \int_1^2 \frac{e^{-\frac{1}{x}}}{x^2} dx = \left[e^{-\frac{1}{x}}\right]_1^2 = \boxed{e^{-\frac{1}{2}} - e^{-1}}$

93. $\int \frac{x}{\sqrt{x^2+3}}\,dx$ let $u = x^2+3$ $du = (2x)\,dx$ $\therefore dx = \frac{du}{2x}$

$= \int \frac{x}{\sqrt{u}} \cdot \frac{du}{2x} = \frac{1}{2}\int u^{-\frac{1}{2}}\,du = \frac{1}{2}\left[\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C_1\right]$

$= u^{\frac{1}{2}} + C_2 = \boxed{\sqrt{x^2+3} + C_2}$ where C_1, C_2 are constants.

94. $\int e^{\ln x}\,dx = \int x\,dx = \boxed{\frac{x^2}{2} + C}$

95. $\int \frac{(x-1)}{(x^2+1)(x+1)}\,dx$ let $\frac{(x-1)}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$

$\therefore (Ax+B)(x+1) + C(x^2+1) = (A+C)x^2 + (A+B)x + (B+C) = 0x^2 + x - 1$

$\therefore \begin{cases} A+C=0 \\ A+B=1 \\ B+C=-1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=0 \\ C=-1 \end{cases}$

$\therefore \int \frac{(x-1)}{(x^2+1)(x+1)}\,dx = \int \frac{x}{x^2+1} + \frac{-1}{x+1}\,dx = \int \frac{x}{x^2+1}\,dx - \int \frac{1}{x+1}\,dx$

$= \int \frac{x}{u} \cdot \frac{1}{2x}\,du - \int \frac{1}{x+1}\,dx$ [let $u = x^2+1$, $du = 2x\,dx$]

$= \frac{1}{2}\ln|u| - \ln|x+1| + C = \boxed{-\ln|1+x| + \frac{1}{2}\ln|1+x^2| + C}$

96. $\int_0^1 \sqrt{1-x^2}\,x^2\,dx$ ~~let $u = 1-x^2$ $\therefore x^2 = 1-u$, $\frac{du}{dx} = -2x$~~

~~$= \int \sqrt{u}\,(1-u)$~~ let $x = \sin\theta$ $\frac{dx}{d\theta} = \cos\theta$

$0 < x < 1 \Rightarrow \theta \in (0, \frac{\pi}{2})$

$\int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\,(\sin^2\theta)\,\cos\theta\,d\theta = \int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\,d\theta = \frac{1}{4}\int_0^{\frac{\pi}{2}} (2\sin\theta\cos\theta)^2\,d\theta$

$= \frac{1}{4}\int_0^{\pi/2} (\sin 2\theta)^2\,d\theta = \frac{1}{4}\int_0^{\pi/2} \frac{1-\cos 4\theta}{2}\,d\theta = \frac{1}{8}\left[\int_0^{\pi/2} 1\,d\theta - \int_0^{\pi/2} \cos 4\theta\,d\theta\right]$

$= \frac{1}{8}\left(\theta - \frac{\sin 4\theta}{4}\right)\Big|_0^{\pi/2}$

$= \boxed{\frac{\pi}{16}}$

97. $\int \cos(x)\ln(\sin(x))\,dx$ $\quad\left[\begin{array}{ll} \text{let } u=\ln(\sin(x)) & du=\frac{1}{\sin(x)}\cdot\cos(x)\,dx \\ e^u=\sin(x) & \end{array}\right.$

$= \int \cos(x)\cdot u\cdot \sin(x)\,\frac{du}{\cos(x)}$

$= \int ue^u\,du$ $\quad\left[\begin{array}{ll} \text{let } m=u & dn=e^u\,du \\ dm=du & n=e^u \end{array}\right]$

$= ue^u - \int e^u\,du$

$= ue^u - e^u + C = (u-1)e^u + C$

$= (\ln(\sin(x))-1)e^{\ln\sin(x)} + C = \boxed{\sin(x)[\ln(\sin(x))-1] + C}$

98. $\int \frac{\sqrt{x}}{x+1}\,dx$ $\quad\left[\begin{array}{ll} \text{let } u=x^{\frac{1}{2}} & du=\frac{1}{2}x^{-\frac{1}{2}}\,dx \\ x=u^2 & dx=2\sqrt{x}\,du \end{array}\right]$

$= \int \frac{\sqrt{x}}{x+1}\cdot 2\sqrt{x}\,du$

$= \int \frac{2x}{x+1}\,du = \int \frac{2u^2}{u^2+1}\,du = \int \frac{2u^2+2-2}{u^2+1}\,du$

$= \int \frac{2(u^2+1)}{u^2+1} + \frac{-2}{u^2+1}\,du$

$= \int 2\,du + (-2)\int \frac{1}{u^2+1}\,du$

$= 2u + (-2)\tan^{-1}(u) + C = \boxed{2\sqrt{x} - 2\tan^{-1}(\sqrt{x}) + C}$

99. $\int \sqrt{\ln x}/x \, dx$ let $z = \sqrt{\ln x}$ $\frac{dz}{dx} = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}$

$= \int \frac{z}{x} \cdot 2xz \cdot dz$ $dx = 2xz \, dz$

$= \int 2z^2 dz = \frac{2}{3} z^3 + C$

$= \frac{2}{3} (\ln x)^{\frac{3}{2}} + C$

100. $\int x^2 e^x dx$

$$\begin{bmatrix} u = x^2 & dv = e^x dx \\ du = 2x & v = e^x \end{bmatrix}$$

$= x^2 e^x - \int e^x \cdot (2x) dx$

$= x^2 e^x - 2\int x e^x dx$

$$\begin{bmatrix} m = x & dv = e^x dx \\ dm = dx & v = e^x \end{bmatrix}$$

$= x^2 e^x - 2[x e^x - \int e^x dx]$

$= x^2 e^x - 2[x e^x - e^x + C_1]$

$= \boxed{e^x [x^2 - 2x + 2] + C_2}$ where C_1, C_2 are constants.

101. Make use of $\sin(3x) = 3\sin x - 4\sin^3(x)$ $\therefore (\sin x)^3 = \frac{3\sin x - \sin(3x)}{4}$

$\therefore \int (\sin x)^3 dx = \int \frac{3\sin(x) - \sin(3x)}{4} dx$

$= \boxed{-\frac{3}{4}\cos(x) + \frac{1}{12}\cos(3x) + C}$

102. $\int \ln(x^2+1)\,dx$

$= x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\,dx$

$\left[\text{let } u = \ln(x^2+1) \quad dv = dx \quad du = \frac{2x}{x^2+1}\,dx \quad v = x\right]$

$= x\ln(x^2+1) - [2x - 2\tan^{-1}(x) + C_1]$ (As in # 98)

$= \boxed{x\ln(x^2+1) - 2x + 2\tan^{-1}(x) + C_2}$

103. $\int \frac{\ln(\ln x)}{x}\,dx$ $\left[\text{let } u = \ln x \quad du = \frac{1}{x}\,dx\right]$

$= \int \frac{\ln(u)}{x} \cdot x\,du$

$= \int \ln(u)\,du$

$\left[\text{let } m = \ln u \quad dn = du \quad dm = \frac{1}{u}\,du \quad n = u\right]$

$= u\ln u - \int u \cdot \frac{1}{u}\,du$

$= u\ln u - u + C \quad = \boxed{(\ln x)\ln(\ln(x)) - \ln(x) + C}$