Problem Set #19 Solutions

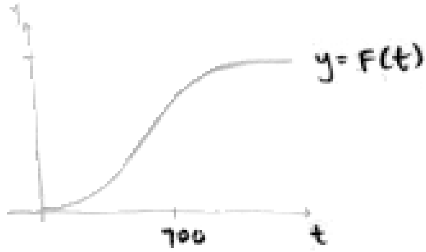
Section 5.10 #53

Problems 2-6 from Handout E

Errors on Worksheet D:

#4 Should be $x=0$ that corresponds to January and not $t=0$

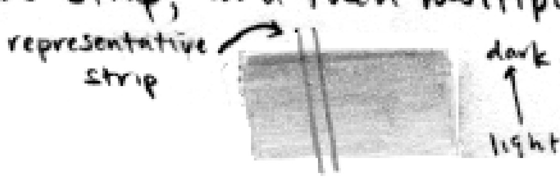
53 a.



b. $r(t) = F'(t)$ This is the rate at which the fraction of burnt-out bulbs increases with time (how $F(t)$ increases as t increases). This is the "fractional burnout rate

c. $\int_0^\infty r(t)\,dt = \lim_{t\to\infty} F(t) = 1$ This is true since we know that eventually all the bulbs will burn out

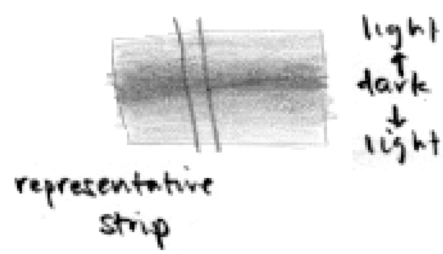
2 a. Note that on a flat surface, mass of cobalt = density $\times$ area However, density changes as a function of x, the distance from the long side of the platter. We can approximate the mass of cobalt used by estimating the mass of cobalt used for a thin strip across the platter where the density of cobalt increases as we move along the strip, and then multiply to get the total mass of cobalt



b. $m = 14\int_0^6 p(x)\,dx$

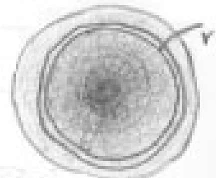
3 a. In a similar way as above we can estimate using a thin strip across the platter and multiply to find the total mass of cobalt used. This time, we examine a strip with highest density at its center and decreasing toward its edges

b. $m = 2\left[14\int_0^3 p(x)\,dx\right]$



④ a. This time we have a circular plate, so instead of using thin strips to estimate, we use thin "washers" (annular rings). These thin washers will have greater density as you move away from the center of the circle

b. $\int_0^8 2\pi p(x)\,dx$



⑤ $\int_{-1}^{1} f(x)\,dx$ can be interpreted as the increase (or decrease) in the population of zebra in the Seronera region of the Serengeti from December through the end of January.

⑥ $\int_0^{10} 4\pi\,(p(x))^2\,dx$