6.7 #2,3,4,6   #2 on K, #1,3 on I   Extra: 6.7 #13

6.7 ② $f(t)$ = Probability Density Function

a) Prob. Drive < 15 min

$$\boxed{\int_0^{15} f(t)\,dt}$$

b) Prob. Drive > 30 min

$$\boxed{\int_{30}^{\infty} f(t)\,dt}$$

6.7 ③ Spinner #s $0 \to 10$

a) Conditions i) $f(x) \geq 0$ for all x

Yes $f(x) = 0$ or 0.1

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Yess! $\int_0^{10} f(x)\,dx = 1$

b) Equal chance for each #
Expected mean = 5

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^{10} x\, f(x)\,dx = \boxed{5}$$

6.7 ④ a) Graph is Prob Density function
Fulfills Both conditions
$f(x) \geq 0$ for all x True

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 = \int_0^{10} f(x)\,dx$$

$$= \underbrace{\tfrac{1}{2} bh}_{\text{Triangle Area}} = \tfrac{1}{2}(10)(0.2) = \boxed{1}$$

b) i $P(x < 3) = \int_0^3 f(x)\,dx = \frac{1}{2}(3)(0.1) = \boxed{0.15}$

ii $P(3 \leq x \leq 8) = \int_3^8 f(x)\,dx$

$$= 1 - P(x \geq 8) - P(x \leq 3)$$

$$= 1 - \tfrac{1}{2}(2)(0.1) - \tfrac{1}{2}(3)(0.1)$$

$$= \boxed{0.75}$$

c) Eqns $f(x) = \begin{cases} \frac{1}{30}x, & 0 \leq x < 6 \\ -\frac{1}{20}x + \frac{1}{2}, & 6 \leq x < 10 \\ 0 & \text{Otherwise} \end{cases}$

$$\text{Mean} = \int_0^6 x\left(\tfrac{1}{30}x\right)dx + \int_6^{10} x\left(-\tfrac{1}{20}x + \tfrac{1}{2}\right)dx$$

$$= \boxed{5.\bar{3}}$$

6.7 ⑥ a) Lightbulb. mean lifetime $\mu = 1000$ hrs
Exponential

$$f(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{1000} e^{-t/1000}, & t \geq 0 \end{cases}$$

i) $P(0 \leq X \leq 200) = \int_0^{200} \frac{1}{1000} e^{-t/1000}\,dt$

$$= \boxed{1 - e^{-1/5}} \approx \boxed{0.181}$$

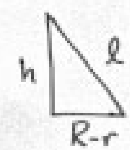
ii) $P(x > 800) = \int_{800}^{\infty} \frac{1}{1000} e^{-t/1000}\,dt = \boxed{0 + e^{-4/5}}$

$$\approx \boxed{0.449}$$

K #2

a) Reasonable Explanation

b) Show by Pythagorean Theorem



$$h^2 + (R-r)^2 = l^2$$

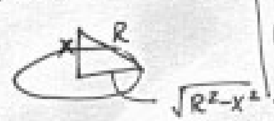
$$l = \sqrt{h^2 + (R-r)^2} = h\sqrt{1 + \left(\frac{R-r}{h}\right)^2}$$

c)

I #1 a) Candy $\rho(x)$ = Calories/mm^3 Calories = $\rho(x)$ (SA) — Surface Area of each shell

$$\#\text{Calories} = \boxed{C = \int_0^R 4\pi x^2 \rho(x)\,dx}$$

b) Bowl



$$\boxed{C = \pi \int_0^R (R^2 - x^2)\, \delta(x)\,dx}$$

Cylindrical Method

Extra 6.7 #13

a) 1st condition $\rho(r) \geq 0$ for $r \geq 0$

2nd $\int_{-\infty}^{\infty} \rho(r)\,dr = \int_0^{\infty} \frac{4}{a_0^3} r^2 e^{-2r/a_0} \underset{\text{Use integration properties, L'Hôpital's Rule}}{=} 1$

b) $\lim_{r \to \infty} \rho(r) = $ w/ L'Hôpital's Rule $= 0$. Max when $\rho'(r) = 0$ when $r = \underline{a_0}$