7.1 #1, 4, 6 + Supplement

7.1 ④ For which r does $y = e^{rt}$ satisfy

$y'' + y' - 6y = 0$

$r^2e^{rt} + re^{rt} - 6e^{rt} = 0$

$r^2 + r - 6 = 0$, $\boxed{r = -3 \text{ or } 2}$

① Show $y = x - \frac{1}{x}$ soln of $xy' + y = 2x$

$y' = 1 + \frac{1}{x^2}$ Plug into ↗

$x\left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right) = 2x + \frac{1}{x} - \frac{1}{x} = 2x$

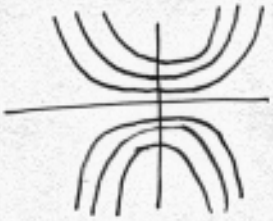
#6

⑥ a) Show $y = Ce^{x^2/2}$ soln to $y' = xy$

$y' = \frac{2x}{2} Ce^{x^2/2} = x \cdot Ce^{x^2/2} = x\left(Ce^{x^2/2}\right)$

Doesn't matter what $C = 0$ Cancels out anyway

b) Graph for diff C values



Sorta ⑨

c) $y' = xy$

$y(0) = 5 = Ce^0 = C = 5$

$\boxed{y = 5e^{x^2/2}}$ soln.

d) $y' = xy$

$y(1) = 2 = Ce^{1/2}$

$C = \frac{2}{\sqrt{e}}$

$y = \boxed{\frac{2}{\sqrt{e}} e^{x^2/2}}$ or $= \boxed{2e^{(x^2-1)/2}}$

7.1 # 3, 9, 12, 14 7.2 #1

7.1 ③ a) $y = \sin kt$ $y' = k\cos kt$ $y'' = -k^2 \sin kt$ Plug in $-k^2\sin kt + 9\sin kt = 0$

$9 - k^2 = 0$, $\boxed{k = \pm 3}$

b) $y = A\sin kt + B\cos kt$, also soln

$y' = Ak\cos kt - Bk\sin kt$

$y'' = -Ak^2\sin kt - Bk^2\cos kt$

Plug in $= 0$ for all $k = \pm 3$ All Values of A, B

7.1 ⑨ a) $\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$

Inc $\frac{dP}{dt} > 0$ when $1 - \frac{P}{4200} > 0$

$\boxed{0 < P < 4200}$

b) Dec $\frac{dP}{dt} < 0$ when $1 - \frac{P}{4200} < 0$

$\boxed{P > 4200}$

c) Equil $\frac{dP}{dt} = 0$ when $1 - \frac{P}{4200} = 0$

$\boxed{P = 4200 \text{ or } P = 0}$

7.1 ⑫ A: $y' = 1 + xy$ $y' < 0$ for Some Points In Quadrant I

B: $y' = -2xy$ $y' = 0$ when $x = 0$

$\boxed{C:}$ Correct $y' = 1 - 2xy$ $x = 0$, $y' = 1$ could work

$x < 0$ $y' > 1$

* Solve $y' = 1 - 2xy$

$y = \frac{1 - y'}{2x}$

as $x \to \infty$, $y \to 0$

⑭ a) Coffee cools Most quickly after removal from source of heat.

Rate of cooling decreases with time

b) Newton's Law $\frac{dP}{dt} = k(M - P)$

c)

always pos, $P \to M$, $\frac{dP}{dt} \to 0$

Supplement Answers

Pg 990 # 1, 2, 4

Money in Account

① a) $\frac{dM}{dt} = 0.04\,M(t)$

b) $\frac{dM}{dt} = 0.04\,M(t) + 1000$

② Disease

$$\frac{dP}{dt} = k\,P(N-P)$$

④ Garbage

$$\frac{dG}{dt} = T - kG$$

312 # 3 (997)

a) $y(t)$ Ce^{kt} soln to $\frac{dy}{dt}$ ky

$\frac{dy}{dt}$ kC^{kt} kCe^{kt} $k(Ce^{kt})$

ky kCe^{kt} True

ke^{t} soln to $\frac{dy}{dt}$ y

$\frac{dy}{dt}$ ke^{t} $ke^{t} \neq k^2e^{t}$

ky k^2e^{t}

c y $e^{kt} + C$ Not soln to $\frac{dy}{dt}$ ky

$\frac{dy}{dt}$ ke^{kt} $ke^{k} \neq k^{kt}\,kC$

$k = ke^{kt} + kC$