Handout J

1) a) $\frac{dx}{dx} = y - x + 1 = 1 + 0 + 1 = \boxed{2}$

b) $h = 1,\ x_0 = 0,\ y_0 = 1,\ F(x,y) = y - x + 1$

$y_1 = y_0 + hF(x_0, y_0) = 1 + (1)(2) = \boxed{3}$

c) $h = 0.5,\ x_0 = 0,\ y_0 = 1,\ F(x,y) = y - x + 1$

$y_1 = y_0 + h\,F(x_0, y_0) = 1 + (0.5)(2) = 2$

$y_2 = y_1 + h\,F(x_1, y_1) = 2 + (0.5)(2 - 0.5 + 1) = \boxed{3.25}$

d) $\frac{dy}{dx} = e^x + 1 \stackrel{?}{=} e^x \cancel{+ x} - x + 1 \qquad y(0) = 1 \checkmark$

$\stackrel{\checkmark}{=} e^x + 1$

$y(1) = e + 1 \approx 3.7183$ which is closer to the answer in c.

2) a) $\frac{dy}{dx} = x^2 + y^2 = 1^2 + 1^2 = \boxed{2}$

b)

c) $y - 1 = 2(x - 1) \quad \boxed{y = 2x - 1}$

d) $\frac{dy}{dx} = x^2 + y^2$

$\frac{d^2y}{dx^2} = 2x + 2y \cdot \frac{dy}{dx} = 2x + 2y(x^2 + y^2)$

at $(1,1)$,

$\frac{d^2y}{dx^2} = 6 > 0$ So, the graph is $\boxed{\text{concave up}}$.

3) The function is never decreasing since the slope is never negative. In fact, since the function can never be equal to 3 (and, therefore, have slope of 0) for $t>0$ because of the initial condition, it is strictly increasing.

$\frac{d^2y}{dx^2} = 2(y-3)\cdot\frac{dy}{dx} = 2(y-3)(y-3)^2$. This is always positive since $y \geq 4$. Therefore, the solution is also concave up for $t>0$.

4) a) i) The function will stay at 0.

ii) The function is strictly increasing since the slope is always positive.

iii) The function will increase until $y=0$ when it will become flat at 0.

b) unstable because a slight change in initial conditions leads to a big change in outcome.

c)
$$\int \frac{dy}{y^2} = \int dx$$
$$\frac{-1}{y} = x + C$$
$$y = \frac{-1}{x+C}$$
$$y(0)=1$$
$$1 = \frac{-1}{C}$$
$$C=-1$$
$$\boxed{y = \frac{-1}{x-1}}$$

d) $\lim_{x \to 1^-} y = +\infty$

5) a) y' should just be 100 since the rate of change remains constant over time.

b) The differential equation is $y' = ky$. The solution is $y = y_0 e^{kt}$.

c) A differential equation represents the rate of change rather than the value of the function.