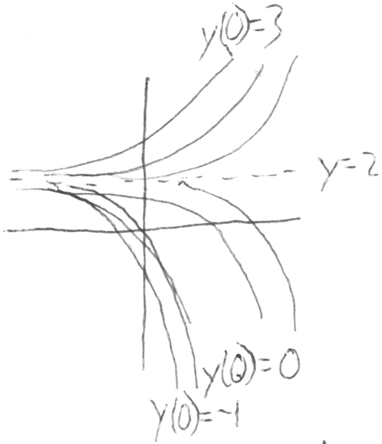
Supp. pg. 14-15 # 1-4

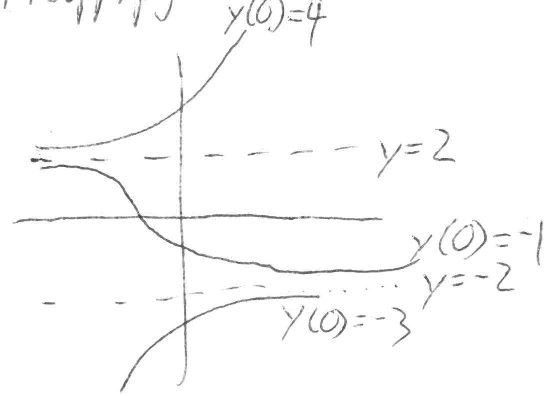
7.2 # 3-6

Extra Credit: Supp. pg. 1000 #17

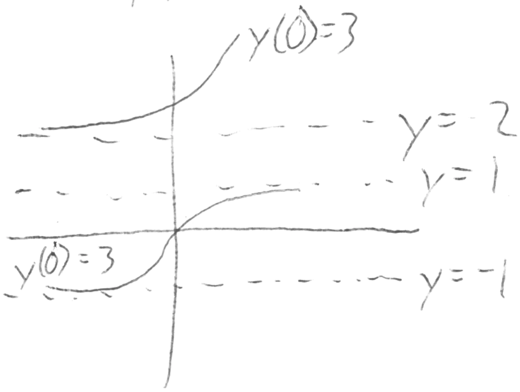
1) a)



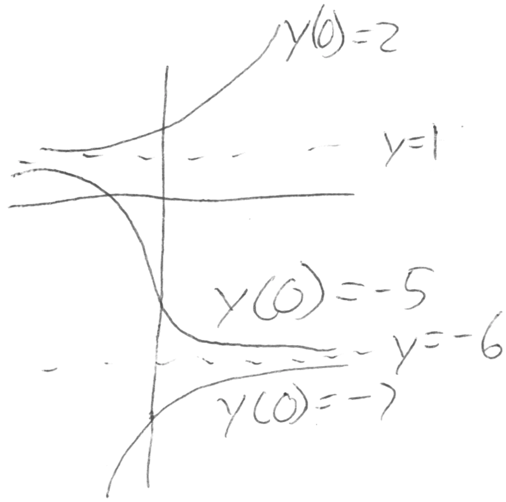
b)



c)



d)



2) a) $\frac{dy}{dt} = 3-y$ b) $\frac{dy}{dt} = y-3$ c) $\frac{dy}{dt} = (2-y)y$ d) $\frac{dy}{dt} = (y-1)(y+1)$

3) a) $y=3$, stable b) $y=3$, unstable c) $y=2$, stable; $y=0$, unstable d) $y=1$, unstable; $y=-1$, stable

4) a) $y=1, 3,$ and 5 which are the zeroes of the graph.

b) When $g(y) > 0$, so $(-\infty, 1) \cup (3, 5)$

7.2

3. $y' = y - 1$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, IV is the direction field for this equation. Note that for $y = 1$, $y' = 0$.

4. $y' = y - x = 0$ on the line $y = x$, when $x = 0$ the slope is y, and when $y = 0$ the slope is $-x$. Direction field II satisfies these conditions. [Looking at the slope at the point $(0, 2)$, II looks more like it has a slope of 2 than does direction field I.]

5. $y' = y^2 - x^2 = 0 \Rightarrow y = \pm x$. There are horizontal tangents on these lines only in graph III, so this equation corresponds to direction field III.

6. $y' = y^3 - x^3 = 0$ on the line $y = x$, when $x = 0$ the slope is y^3, and when $y = 0$ the slope is $-x^3$. The graph is similar to the graph for Exercise 4, but the segments must get steeper very rapidly as they move away from the origin, because x and y are raised to the third power. This is the case in direction field I.

Supp. pg. 1000 #17
extra credit

17) a) $\frac{dP}{dt} = kP - E$

$$\int \frac{dP}{kP-E} = \int dt$$

$$\frac{\ln(kP-E)}{k} = t + C$$

$$\ln(kP-E) = kt + kC$$

$$kCe^{kt} = kP - E$$

$$Ce^{kt} = P - \frac{E}{k}$$

$$\boxed{P = Ce^{kt} + \frac{E}{k}}$$

b) $\frac{dP}{dt} = kCe^{kt}$

$$kCe^{kt} \stackrel{?}{=} k\left(Ce^{kt} + \frac{E}{k}\right) - E$$

$$\stackrel{?}{=} kCe^{kt} + E - E$$

$$\stackrel{\checkmark}{=} kCe^{kt}$$

So, this is the solution.

7.3

3. $yy' = x \;\Rightarrow\; y\dfrac{dy}{dx} = x \;\Rightarrow\; \int y\,dy = \int x\,dx \;\Rightarrow\; \frac{1}{2}y^2 = \frac{1}{2}x^2 + C_1 \;\Rightarrow\; y^2 = x^2 + 2C_1 \;\Rightarrow$
$x^2 - y^2 = C$ (where $C = -2C_1$). This represents a family of hyperbolas.

4. $y' = xy \;\Rightarrow\; \displaystyle\int \frac{dy}{y} = \int x\,dx \quad [y \neq 0] \;\Rightarrow\; \ln|y| = \frac{x^2}{2} + C \;\Rightarrow\; |y| = e^C e^{x^2/2} \;\Rightarrow\; y = Ke^{x^2/2}$,
where $K = \pm e^C$ is a constant. (In our derivation, K was nonzero, but we can restore the excluded case $y = 0$ by allowing K to be zero.)

5. $\dfrac{dy}{dt} = \dfrac{te^t}{y\sqrt{1+y^2}} \;\Rightarrow\; y\sqrt{1+y^2}\,dy = te^t\,dt \;\Rightarrow\; \int y\sqrt{1+y^2}\,dy = \int te^t\,dt \;\Rightarrow$
$\frac{1}{3}(1+y^2)^{3/2} = te^t - e^t + C$ [where the first integral is evaluated by substitution and the second by parts] $\Rightarrow$
$1 + y^2 = [3(te^t - e^t + C)]^{2/3} \;\Rightarrow\; y = \pm\sqrt{[3(te^t - e^t + C)]^{2/3} - 1}$

6. $y' = \dfrac{xy}{2\ln y} \;\Rightarrow\; \dfrac{2\ln y}{y}\,dy = x\,dx \;\Rightarrow\; \displaystyle\int \frac{2\ln y}{y}\,dy = \int x\,dx \;\Rightarrow\; (\ln y)^2 = \frac{x^2}{2} + C \;\Rightarrow$
$\ln y = \pm\sqrt{x^2/2 + C} \;\Rightarrow\; y = e^{\pm\sqrt{x^2/2+C}}$

9. $\dfrac{dy}{dx} = y^2 + 1,\ y(1) = 0.\ \displaystyle\int \frac{dy}{y^2+1} = \int dx \;\Rightarrow\; \tan^{-1} y = x + C.$ $y = 0$ when $x = 1$, so
$1 + C = \tan^{-1} 0 = 0 \;\Rightarrow\; C = -1$. Thus, $\tan^{-1} y = x - 1$ and $y = \tan(x-1)$.

16. $\dfrac{dy}{dx} = \dfrac{y^2}{x^3},\ y(1) = 1.\ \displaystyle\int \frac{dy}{y^2} = \int \frac{dx}{x^3} \;\Rightarrow\; -\frac{1}{y} = -\frac{1}{2x^2} + C.\ y(1) = 1 \;\Rightarrow\; -1 = -\frac{1}{2} + C \;\Rightarrow$
$C = -\frac{1}{2}$. So $\dfrac{1}{y} = \dfrac{1}{2x^2} + \dfrac{1}{2} = \dfrac{2+2x^2}{2\cdot 2x^2} \;\Rightarrow\; y = \dfrac{2x^2}{x^2+1}$.

28. From Exercise 7.2.28, $\dfrac{dy}{dt} = -\frac{1}{50}(y - 20) \;\Leftrightarrow\; \displaystyle\int \frac{dy}{y-20} = \int \left(-\tfrac{1}{50}\right) dt \;\Leftrightarrow\; \ln|y - 20| = -\tfrac{1}{50}t + C \;\Leftrightarrow$
$y - 20 = Ke^{-t/50} \;\Leftrightarrow\; y(t) = Ke^{-t/50} + 20.\ y(0) = 95 \;\Leftrightarrow\; 95 = K + 20 \;\Leftrightarrow\; K = 75 \;\Leftrightarrow$
$y(t) = 75e^{-t/50} + 20$.

33. (a) $\dfrac{dC}{dt} = r - kC \;\Rightarrow\; \dfrac{dC}{dt} = -(kC - r) \;\Rightarrow\; \displaystyle\int \frac{dC}{kC - r} = \int -dt \;\Rightarrow\; (1/k)\ln|kC - r| = -t + M_1$
$\Rightarrow\; \ln|kC - r| = -kt + M_2 \;\Rightarrow\; |kC - r| = e^{-kt+M_2} \;\Rightarrow\; kC - r = M_3e^{-kt} \;\Rightarrow$

$kC = M_3e^{-kt} + r \;\Rightarrow\; C(t) = M_4e^{-kt} + r/k.\ C(0) = C_0 \;\Rightarrow\; C_0 = M_4 + r/k \;\Rightarrow$
$M_4 = C_0 - r/k \;\Rightarrow\; C(t) = (C_0 - r/k)e^{-kt} + r/k$.

(b) If $C_0 < r/k$, then $C_0 - r/k < 0$ and the formula for $C(t)$ shows that $C(t)$ increases and $\lim_{t\to\infty} C(t) = r/k$.
As t increases, the formula for $C(t)$ shows how the role of C_0 steadily diminishes as that of r/k increases.