Mathematics 1b - Solution Set for PS 2
Problem Set # 2

Do: §8.2 # 12, 16, 18, 20, 33, 34, 48, 51

12) $1 + 0.4 + 0.16 + 0.064...$ is a geometric series with ratio 0.4. The series converges because $|r| < 1$. The sum converges to $\frac{a}{1-r} = \frac{1}{1-2/5} = \frac{5}{3}$.

16) $\sum_{n=1}^{\infty}(\frac{1}{e^2})^n$ converges because $|r| = \frac{1}{e^2} < 1$. a =first term= $\frac{1}{e^2}$. Series converges to $\frac{1/e^2}{1-1/e^2} = \frac{1}{e^2-1}$

18) $\sum_{n=1}^{\infty}\frac{3}{n} = 3 * \sum_{n=1}^{\infty}\frac{3}{n}$, which diverges because its partial sums are 3 times the corresponding partial sum of the harmonic seris $\sum_{n=1}^{\infty}\frac{3}{n}$, which diverges.

20) $\sum_{n=1}^{\infty}(\frac{(n+1)^2}{n(n+2)})$ diverges because of the Test for Divergence.
$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n^2+2n+1}{n^2+2n} = 1$, not 0.

33) $\sum_{n=1}^{\infty}\frac{x^n}{3^n} = \sum_{n=1}^{\infty}(\frac{x}{3})^n$. This geometric series converges when $|r| = |\frac{x}{3}| < 1$. This occurs when $-3 < x < 3$ When x is in this interval, the sum converges to $\frac{x/3}{1-x/3} = \frac{x}{3-x}$.

34) $\sum_{n=0}^{\infty}(2^n(x+1)^n) = \sum_{n=0}^{\infty}[2(x+1)]^n$.
$r = 2(x+1)$, so the series converges if $|r| < 1$:
$|2(x+1)| < 1 \longrightarrow |2x+2| < 1 \longrightarrow -1 < 2x+2 < 1 \longrightarrow -3 < 2x < -1 \longrightarrow -\frac{3}{2} < x < -\frac{1}{2}$.
The sum would be $\frac{1}{1-2(x+1)} = \frac{1}{-1-2x} = \frac{-1}{2x+1}$.

48) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$ by Theorem 6, so $\lim_{n\to\infty}\frac{1}{a_n}$ cannot be zero. Thus, $\sum_{n=1}^{\infty}\frac{1}{a_n}$ is divergent by the Test for Divergence.

51) The partial sums s_n form an increasing sequence, since $s_n - S_{n-1} = a_n > 0$ for all n. Also, the sequence s_n is bounded since $s_n \leq 1000$ for all n. So by the Monotonic Sequence Theorem, the sequence of partial sums converges and the series is convergent.

Think, and think again: Write out the first few terms of the series $\sum_{k=1}^{\infty}(\frac{1}{2^k+k})$. (This series is not a geometric series.) Now write out the first few terms of the geometric series $\sum_{k=1}^{\infty}(\frac{1}{2^k})$. By comparing the terms of the two series, determine whether or not the former series converges. Explain your reasoning in words carefully and clearly. Your answer will form the launching pad for the next class.

$\sum_{k=1}^{\infty}(\frac{1}{2^k+k}) = \frac{1}{3} + \frac{1}{6} + \frac{1}{11} + \frac{1}{20} + ...$
$\sum_{k=1}^{\infty}(\frac{1}{2^k}) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$
In general, $\frac{1}{2^k} > \frac{1}{2^k+k}$ for any $k > 0$. Thus, $\sum_{k=1}^{\infty}(\frac{1}{2^k+k})$ must converge because $\sum_{k=1}^{\infty}(\frac{1}{2^k})$ is a geometric with $r = \frac{1}{2} < 1$ and with partial sums that are always greater than corresponding partial sums of the non-geometric series. Thus, if the geometric converges and both series are constantly increasing (as these both are), the non-geometric series must also converge.

Extra credit # 52.

52) a) $\frac{1}{f_{n-1}f_n} - \frac{1}{f_nf_{n+1}} = \frac{f_nf_{n+1} - f_nf_{n-1}}{f_n^2f_{n-1}f_{n+1}} = \frac{f_{n+1}-f_{n-1}}{f_nf_{n-1}f_{n+1}} = \frac{(f_{n-1}+f_n)-f_{n-1}}{f_nf_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_{n+1}}$

b) $\sum_{n=2}^{\infty}(\frac{1}{f_{n-1}f_{n+1}}) = \sum_{n=2}^{\infty}(\frac{1}{f_{n-1}f_n} - \frac{1}{f_nf_{n+1}}) = \lim_{n\to\infty}((\frac{1}{f_1f_2} - \frac{1}{f_2f_3}) + (\frac{1}{f_2f_3} - \frac{1}{f_3f_4}) + (\frac{1}{f_3f_4} - \frac{1}{f_4f_5}) + ... + (\frac{1}{f_{n-1}f_n} - \frac{1}{f_nf_{n+1}}))$

$= \lim_{n\to\infty}(\frac{1}{f_1f_2} - \frac{1}{f_nf_{n+1}} = \frac{1}{f_1f_2} - 0 = \frac{1}{1*1} = 1$

c) $\sum_{n=2}^{\infty}\frac{f_n}{f_{n-1}f_{n+1}} = \sum_{n=2}^{\infty}(\frac{f_n}{f_{n-1}f_n} - \frac{f_n}{f_nf_{n+1}}) = \sum_{n=2}^{\infty}(\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}}) = \lim_{n\to\infty}[(\frac{1}{f_1} - \frac{1}{f_3}) + (\frac{1}{f_2} - \frac{1}{f_4}) + (\frac{1}{f_3} - \frac{1}{f_5}) + (\frac{1}{f_4} - \frac{1}{f_6}) + ... + (\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}})]$

$\lim_{n\to\infty}(\frac{1}{f_1} + \frac{1}{f_2} - \frac{1}{f_n} - \frac{1}{f_{n+1}}) = 1 + 1 - 0 - 0 = 2.$