Mathematics 1b - Solution Set for PS 0

1 Primer for Integration

(a) $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C \qquad (n \neq -1)$
(b) $\int \frac{1}{u}\,du = \ln u + C$
(c) $\int \sin u\,du = -\cos u + C$
(d) $\int \cos u\,du = \sin u + C$
(e) $\int \sec^2 u\,du = \tan u + C$
(f) $\int e^u\,du = e^u + C$
(g) $\int b^u\,du = \frac{1}{\ln b}b^u + C$
(h) $\int \frac{1}{1+u^2}\,du = \arctan u + C$
(i) $\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin u + C$

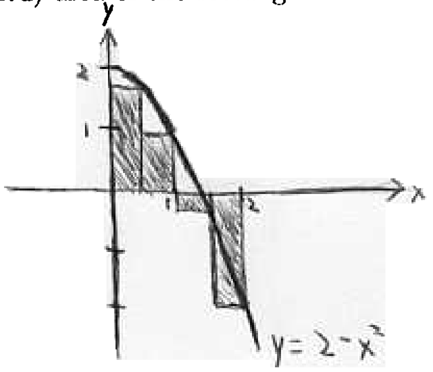
2. (a) i) $\int (2x+1)^3\,dx = \frac{1}{8}(2x+1)^4 + C$ (the appropriate substitution is $u = 2x+1$)
   ii) $\int \frac{1}{(2x+1)^3}\,dx = -\frac{1}{4}\frac{1}{(2x+1)^2} + C$ $(u = 2x+1)$
   iii) $\int \frac{1}{2x+1}\,dx = \frac{1}{2}\ln|2x+1| + C$ $(u = 2x+1)$
   (b) i) $\int x\sqrt{x^2+5}\,dx = \frac{1}{3}(x^2+5)^{3/2} + C$ $(u = x^2+5)$
   ii) $\int \sqrt{\cos x}\sin x\,dx = -\frac{2}{3}(\cos x)^{3/2} + C$ $(u = \cos x)$
   iii) $\int t^2\sin(t^3)\,dt = -\frac{1}{3}\cos t^3 + C$ $(u = t^3)$
   iv) $\int \tan t\,dt = \ln|\sec t| + C$ $(u = \cos t)$
   (c) i) $\int \frac{\ln x}{x}\,dx = \frac{1}{2}(\ln x)^2 + C$ $(u = \ln x)$
   ii) $\int \frac{e^x}{e^{-x}}\,dx = \frac{1}{2}e^{2x} + C$ $(u = 2x)$
   iii) $\int \frac{x}{e^{x^2}}\,dx = -\frac{1}{2}e^{-x^2} + C$ $(u = x^2)$

3. <u>5.2.1</u>

Dividing the interval of length 2 into four equal subintervals tells us that $\Delta x = \frac{1}{2}$. The right endpoints of the intervals are $x_0 = \frac{1}{2}, x_1 = 1, x_2 = \frac{3}{2}$, and $x_3 = 2$. The corresponding values of the function at those points are $f(x_0) = \frac{7}{4}, f(x_1) = 1, f(x_2) = -\frac{1}{4}$, and $f(x_3) = -2$. The Riemann sum is then:

$$\sum_{i=0}^{n-1} f(x_i)\Delta x = \left(\frac{7}{4}\right)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}\right) + \left(-\frac{1}{4}\right)\left(\frac{1}{2}\right) + (-2)\left(\frac{1}{2}\right) = \frac{1}{4}$$

This sum represents the total (signed) area of the rectangles in this diagram:



5.2.7

Again we use the idea of the Riemann sum: since we are only given values of the function at intervals of 5 units, we set $\Delta x = 5$. Furthermore, the function is increasing, so to get a lower estimate, we should let our x_i^*'s be the left endpoints of the intervals. This yields the following:

$$\sum_{i=0}^{n-1} f(x_i^*)\Delta x = (-42 \cdot 5) + (-37 \cdot 5) + (-25 \cdot 5) + (-6 \cdot 5) + (15 \cdot 5) = -475$$

(Note that though there are 6 values of x for which $f(x)$ is specified, this only gives us 5 intervals.) Similarly, to obtain an upper estimate, we let our x_i's be the right endpoints of the intervals:

$$\sum_{i=0}^{n-1} f(x_i^*)\Delta x = (-37 \cdot 5) + (-25 \cdot 5) + (-6 \cdot 5) + (15 \cdot 5) + (36 \cdot 5) = -85$$

5.2.30c

$$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = \frac{1}{2} \cdot 2 \cdot 4 + (-\frac{1}{2} \cdot \pi \cdot 2^2) + \frac{1}{2} \cdot 1 \cdot 1 = \frac{9}{2} - 2\pi$$

CC 2

(a) $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i^*)\Delta x$, where $\Delta x = \frac{b-a}{n}$ and x_i^* is a sample point in the subinterval $[a + i\Delta x, a + (i+1)\Delta x]$.
(b) It is the area below the curve (and above the x-axis).
(c) This is the signed area of what is between the curve and the x-axis; areas above the x-axis are a positive contribution, while areas below the x-axis are a negative contribution.

CC 5

(a) It is the displacement, in feet, of the particle between $t = 60$ seconds and $t = 120$ seconds.
(b) It is the total distance traveled, in feet, by the particle between $t = 60$ seconds and $t = 120$ seconds.
(c) It is the change in velocity, in feet per second, of the particle between $t = 60$ seconds and $t = 120$ seconds.

2 Graphing Primer

1. $y = f(x-2)$ is $y = f(x)$ shifted to the right by 2 units. The roots of $f(x-2)$ are at $x = 5, 9$, and 12.

2. Polynomials are the sum of a finite number of coefficients times a variable raised to nonzero integer values; the exponential, trigonometric, and logarithmic functions are not. Graphs of polynomials cannot have asymptotes; the graphs of exponential, logarithmic, and certain trigonometric functions do have asymptotes. Both the derivative and the antiderivative of any polynomial must again be polynomials; conversely, polynomials can never be the derivatives nor antiderivatives of any (non-trivial) exponential, trigonometric, or logarithmic functions.

3. Note that if a point is indicated as being on the graph of a function (e.g., for graph I, $(-3, 0)$ is on the function), it must satisfy any candidate equation for the graph. This is still not enough to specify completely the equation for each of the graphs, but the "best," i.e., simplest, answers are shown below.

   I. $y = -\frac{1}{3}x^2 + 3$
   II. $y = 2 \cdot 2^{-x}$ (the 2 in the base of the power can be varied, but the other 2 cannot be)
   III. $y = \ln(x+1)$
   IV. $y = -2 \cos x$
   V. $y = \cos(\pi x) + 1$
   VI. $y = 1 - 10^{-x}$ (the 10 can be varied)
   VII. $y = \sin(x^2)$
   VIII. $y = 10^{-\frac{x}{10}} \sin x$ (the constants can be varied)