

5.10

$$24) \int_0^3 \frac{dx}{x\sqrt{x}} = \lim_{t \rightarrow 0^+} \int_t^3 \frac{dx}{x^{3/2}} = \lim_{t \rightarrow 0^+} \left[ \frac{-2}{\sqrt{x}} \right]_t^3 = \frac{-2}{\sqrt{3}} + \lim_{t \rightarrow 0^+} \frac{2}{\sqrt{t}} = \infty$$

Divergent

$$29) \int_{-2}^3 \frac{dx}{x^4} = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}, \text{ but } \int_{-2}^0 \frac{dx}{x^4} = \lim_{t \rightarrow 0^+} \left[ -\frac{x^{-3}}{3} \right]_{-2}^{-t} =$$

$$\lim_{t \rightarrow 0^+} \left[ -\frac{1}{3t^3} - \frac{1}{24} \right] = \infty$$

Divergent

6.7

#2

a) The probability is  $\int_0^{15} f(t) dt$

$$b) \int_{30}^{\infty} f(t) dt$$

#4 a)  $f(x) \geq 0$

$$\int_{-\infty}^{\infty} f(x) dx = \int_0^{10} f(x) dx = \frac{1}{2} (10)(2) = 1 \quad f(x)$$

is a probability density function

$$b) (i) P(X < 3) = \int_0^3 f(x) dx = \frac{1}{2} (3)(.1) = \frac{3}{20} = .15$$

$$(ii) P(X > 8) = \int_8^{10} f(x) dx = \frac{1}{2} (2)(.1) = \frac{2}{20} = (.1)$$

$$P(3 \leq X \leq 8) = 1 - .15 - .10 = .75$$

6)

$$a) \mu = 1000 \Rightarrow f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{1000} e^{-\frac{t}{1000}} & t \geq 0 \end{cases}$$

$$(c) P(0 \leq X \leq 200) = \int_0^{200} \frac{1}{1000} e^{-\frac{t}{1000}} dt = \left[ -e^{-\frac{t}{1000}} \right]_0^{200} =$$

$$e^{-\frac{1}{5}} + 1 \approx .171$$

$$P(X > 800) = \int_{800}^{\infty} \frac{1}{1000} e^{-\frac{t}{1000}} dt = \lim_{x \rightarrow \infty} \left[ -e^{-\frac{t}{1000}} \right]_{800}^x$$

$$0 + e^{-\frac{4}{5}} \approx .449$$

$$b) \text{ find } m \text{ s.t. } \int_m^{\infty} f(t) dt = \frac{1}{2} \quad \lim_{x \rightarrow \infty} \int_m^x \frac{1}{1000} e^{-\frac{t}{1000}} = \frac{1}{2}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left[ -e^{-\frac{t}{1000}} \right]_m^{\infty} = \frac{1}{2} \Rightarrow 0 + e^{-\frac{m}{1000}} = \frac{1}{2}$$

$$\Rightarrow -\frac{m}{1000} = \ln \frac{1}{2} \Rightarrow m = -1000 \ln \left( \frac{1}{2} \right) = 1000 \ln 2 = 693.1$$