PS 12 DJ

6.4

2) $g_{ave} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{2}{9}\left[x^{3/2}\right]_1^4 = \frac{2}{9}[8-1] = \frac{14}{9}$

11) Let $t=0$ and $t=12$ correspond to 9 A.M and 9 P.M. respectively

$$T_{ave} = \frac{1}{12-0}\int_0^{12}\left[50 + 14\sin\left(\tfrac{1}{12}\pi t\right)\right]dt = \frac{1}{12}\left[50t - 14\cdot\tfrac{12}{\pi}\cos\left(\tfrac{1}{12}\pi t\right)\right]_0^{12}$$

$$= \frac{1}{12}\left[50\cdot 12 + 14\cdot\tfrac{12}{\pi} + 14\cdot\tfrac{12}{\pi}\right] = \left(50 + \tfrac{28}{\pi}\right)^{\circ}F \approx \underline{59^{\circ}F}$$

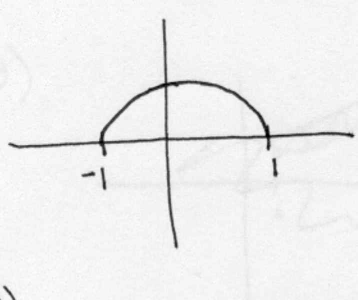
6.3

4) $y = 2^x \qquad \frac{dy}{dx} = 2^x \ln 2 \Rightarrow L = \int_0^3 \sqrt{1 + (\ln 2)^2\, 2^{2x}}\,dx$

Integration Handout

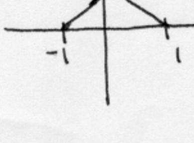
23) $\int_{-1}^{1} f(x)\,dx$ gives us the change in the number of zebra in the Seronera region of the Serengeti between December and February.

24) a) $\sqrt{1-x^2}$



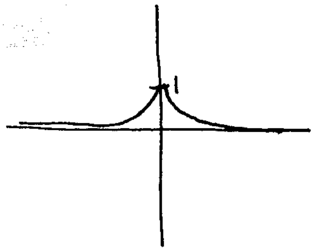
It appears that the function goes between 0 and 1 giving the approx avg. value of $\boxed{.75}$

b) $-|x|+1$



This appears to be linear on both sides, thus we would estimate $\boxed{.5}$

c) $e^{-|x|}$



This should be higher than (b), but less than (a) thus $\boxed{1.65}$.

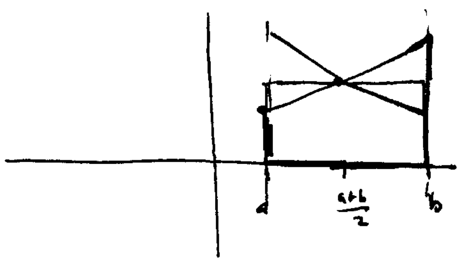
25) a) If $f''(x)=0$, f is linear

$f'(x)=c_1$ on $[a,b]$

$f(x)=c_1x+c_2$

$$\frac{(c_1 b+c_2)+(c_1 a+c_2)}{2} = \frac{c_1(b+a)+2c_2}{2}$$
$$= c_1\left(\frac{a+b}{2}\right)+c_2$$
$$= f\left(\frac{a+b}{2}\right)$$

b)



Area of rectangle

$f\left(\frac{a+b}{2}\right)(b-a)$

Area of trapezoid

$\frac{1}{2}(b_1+b_2)h$

$\frac{1}{2}(f(a)+f(b))(b-a)$

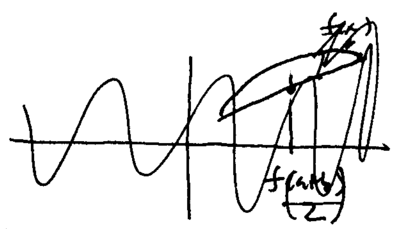
$f\left(\frac{a+b}{2}\right)(b-a)$ (a) since

$\frac{1}{2}(f(a)+f(b)) = f\left(\frac{a+b}{2}\right)$

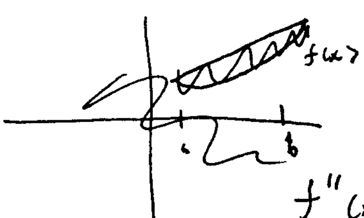
c) $f''(x)>0$

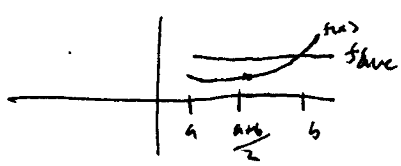
f is convex, thus on any interval $[a,b]$

$f_{ave} > f\left(\frac{a+b}{2}\right)$



d)





$f''(x)<0$

f is concave $f_{ave} < f\left(\frac{a+b}{2}\right)$