Problem Set 17 Solutions

30.1

① $f(x) = e^{-x}$

$$f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^4(0)x^4}{4!}$$

$f(0) = 1$

$f'(0) = -e^{-(0)} = -1$

$f''(0) = e^{-(0)} = 1$

$f'''(0) = -e^{-(0)} = -1$

$f^4(0) = e^{-(0)} = 1$

$$\boxed{P_4(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}}$$

Graph on Calculator

$P_1(.1) = .9$ $\quad P_1(.3) = .7$

$P_2(.1) = .905$ $\quad P_2(.3) = .745$

$P_3(.1) = .904\bar{6}$ $\quad P_3(.3) = .736$

$P_4(.1) = .90467$ $\quad P_4(.3) = .73634$

$e^{-.1} = .9048$ $\quad e^{-.3} = .74082$

② $f(x) = \ln(1+x)$

$$f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^4(0)x^4}{4!}$$

$f(0) = 0$

$f'(x) = \frac{1}{1+x} = \frac{1}{1+0} = 1$

$f''(x) = \frac{-1}{(1+x)^2} = \frac{-1}{(1+0)^2} = -1$

$f'''(x) = \frac{2}{(1+x)^3} = \frac{2}{(1+0)^3} = 2$

$f^4(x) = \frac{-6}{(1+x)^4} = \frac{-6}{(1+0)^4} = -6$

$$P_4(x) = x - \frac{x^2}{2} + \frac{2x^3}{6} - \frac{6x^4}{24} = \boxed{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}}$$

$P_1(.1) = .1$ $\quad P_1(.3) = .3$

$P_2(.1) = .095$ $\quad P_2(.3) = .255$

$P_3(.1) = .095\overline{3}$ $\quad P_3(.3) = .264$

$P_4(.1) = .09531$ $\quad P_4(.3) = .26198$

$f(.1) = .09531$ $\quad f(.3) = .26236$

(9) $P_2(0) = f(0)$

$P_2'(0) = f'(0)$

$P_2''(0) = f''(0)$

a_0 – negative

a_1 – positive

a_2 – positive

(11) a) $f(x) = \tan(x)$

$$f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{6}$$

$f(0) = 0$

$$f'(x) = \frac{1}{\cos^2 x} \rightarrow \frac{1}{\cos^2(0)} = 1$$

$$f''(x) = \frac{2\sin x}{\cos^3 x} \rightarrow \frac{2\sin(0)}{\cos^3(0)} = 0$$

$$f'''(x) = \frac{4\sin^2 x + 2}{\cos^4 x} \rightarrow \frac{4\sin^2(0) + 2}{\cos^4(0)} = 2$$

$$\boxed{P_3(x) = x + \frac{x^3}{3}}$$

b) Tan is an odd function, and thus would not have terms with even exponents.

(12) a) a_0 - negative
a_1 - zero
a_2 - positive

b) a_0 - negative
a_1 - positive
a_2 - positive

c) a_0 - zero
a_1 - positive
a_2 - zero

d) a_0 - positive
a_1 - positive
a_2 - negative

(13) from #2

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + \frac{(-1)^{n+1} x^n}{n}$$

(16) $\sqrt{103} \approx \sqrt{100} \rightarrow \sqrt{x} = f(x)$

$$f(100) + f'(100)(x-100) + \frac{f''(100)(x-100)^2}{2}$$

$f(100) = 10$

$f'(x) = \frac{1}{2\sqrt{x}} \rightarrow \frac{1}{2\sqrt{100}} = .05$

$f''(x) = \frac{-1}{4x^{3/2}} \rightarrow \frac{-1}{4(100)^{3/2}} = \frac{-1}{4000}$

$P_2(x) = 10 + .05(x-100) - \frac{.00025(x-100)^2}{2}$

$P_2(103) = 10 + .05(103-100) - \frac{.00025(103-100)^2}{2}$

$\boxed{P_2(103) = 10.148875}$

Series Handout #10

(10) a) $P_1(x) = 4x$

$P_2(x) = 4x - 4x^2$

$P_3(x) = 4x - 4x^2 + x^3$

The best linear approximation is the tangent line at $b=0$. This reduces to $y=$

The approximation is not surprising; the best polynomial approximation of a polynomial is itself.

b) a_1 is defined as the derivative of $f(x)$ [divided by $1! = 1$]. At $x=2$, however the derivative of $f(x)$ is zero since it is a critical point. Thus $a_1 = 0$.

$a_4 \to a_n$ will all be zero because the original function is cubic; it makes no sense to approximate further.

c) $a_4, a_5 \to a_n$ will still all be zero as above.