PSET 26

1. a) $\frac{dx}{dt} = -2t$ $\quad x = -t^2 + c$ $\quad x(0) = 1 \Rightarrow x = -t^2 + 1$

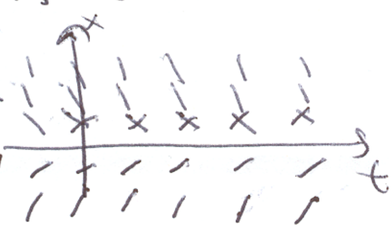
$x(0) = 0 \Rightarrow x = -t^2$

$x(0) = -1 \Rightarrow x = -t^2 - 1$

b) $\frac{dx}{dt} = -2x$ $\quad \cancel{t}\ x = e^{-2t} \cdot c$ $\quad x(0) = 1 \Rightarrow x = e^{-2t} \cdot 1$

$x(0) = 0 \Rightarrow x = e^{-2t} \cdot 0 = 0$

$x(0) = -1 \Rightarrow x = e^{-2t} \cdot (-1)$



c) $\frac{dx}{dt} = t^2$ $\quad x = \frac{t^3}{3} + c$ $\quad x(0) = 1 \Rightarrow x = \frac{t^3}{3} + 1$

$x(0) = 0 \Rightarrow x = \frac{t^3}{3} + 0$

$x(0) = -1 \Rightarrow x = \frac{t^3}{3} - 1$

None of the solutions intersect.

2) Only (d) is a soln: $\frac{dy}{dt} = ce^t + 0 = y + 1$

a, b, c and e are not solutions

3) Only (e) is a solution $\frac{dy}{dt} = -3 \cdot 5 \sin(3t)$ $\quad \frac{d^2y}{dt^2} = -9 \cdot 5 \cos(3t) = -9y.$

4) $\frac{dC}{dt} = k(L - C(t))$ for some constant k. k is positive, because if $L > C(t)$, solute should flow into the cell.

5) a) $dI/dt = k(N - I)I$ where k is positive because people can only become infected, not recover.

b) The number is increasing as long as $0 \leq I \leq N$, which is neccessary as I is a subset of the population. Eventually, everyone gets sick.

3. $y' = y - 1$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, IV is the direction field for this equation. Note that for $y = 1$, $y' = 0$.

4. $y' = y - x = 0$ on the line $y = x$, when $x = 0$ the slope is y, and when $y = 0$ the slope is $-x$. Direction field II satisfies these conditions. [Looking at the slope at the point $(0, 2)$, II looks more like it has a slope of 2 than does direction field I.]

5. $y' = y^2 - x^2 = 0 \quad \Rightarrow \quad y = \pm x$. There are horizontal tangents on these lines only in graph III, so this equation corresponds to direction field III.

6. $y' = y^3 - x^3 = 0$ on the line $y = x$, when $x = 0$ the slope is y^3, and when $y = 0$ the slope is $-x^3$. The graph is similar to the graph for Exercise 4, but the segments must get steeper very rapidly as they move away from the origin, because x and y are raised to the third power. This is the case in direction field I.