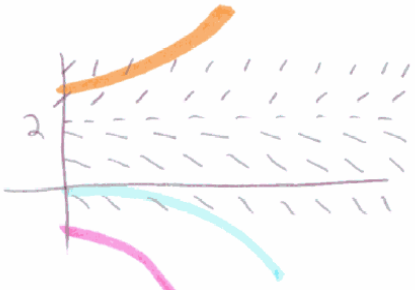
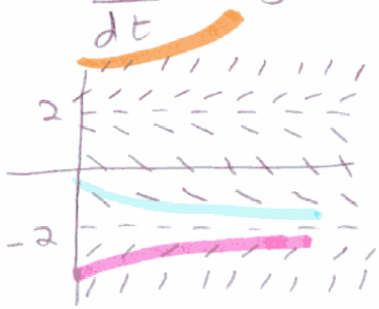
Math 1b: Problem Set 28

1. a) $\frac{dy}{dt} = 4y - 8$ $\quad \frac{dy}{dt} = 0$ at $y = 2$



$y(0) = 0$
$y(0) = -1$
$y(0) = 3$

b) $\frac{dy}{dt} = y^2 - 4 = (y+2)(y-2)$ $\quad \frac{dy}{dt} = 0$ at $y = \pm 2$

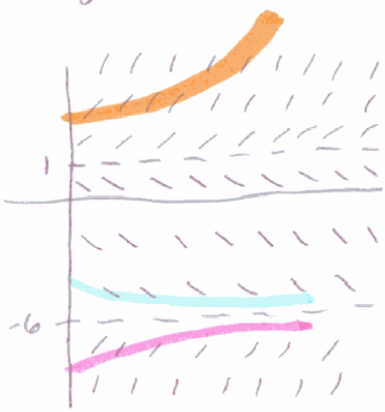


$y(0) = -1$
$y(0) = -3$
$y(0) = 4$

c) $\frac{dy}{dt} = (y-1)(y-2)(y+1)$

$y(0) = 0$
$y(0) = 3$

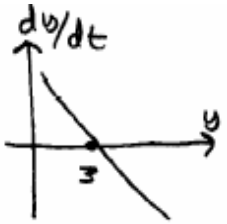
d) $\frac{dy}{dt} = y^2 + 5y - 6 = (y+6)(y-1)$



$y(0) = -5$
$y(0) = -7$
$y(0) = 2$

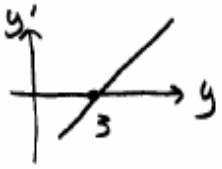
2. a) $\frac{dy}{dt} > 0$ for $y < 3$

$\frac{dy}{dt} < 0$ for $y > 3$

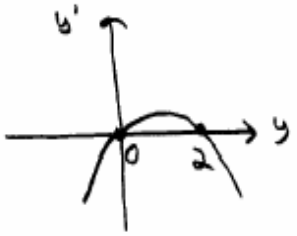


$$\frac{dy}{dt} = 3 - y$$

b) $\frac{dy}{dt} = y - 3$

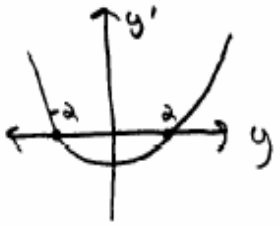


c)



$$\frac{dy}{dt} = -y(y-2)$$

d)



$$\frac{dy}{dt} = (y-2)(y+2)$$

3) a) $y = 3$ is stable

b) $y = 3$ is unstable

c) $y = 2$ is stable

$y = 0$ is unstable

d) $y = 2$ is unstable

$y = -2$ is stable

6. $\frac{dy}{dt} = y^2 - 1 = (y+1)(y-1)$

Equilibrium points at $y = 1$ and $y = -1$, so a, b, e are out. $\left.\frac{dy}{dt}\right|_{y=0} = -1 < 0$, so (d) is possible

7.4 #19bcd)

(b) There will be an exponential expansion $\Leftrightarrow$ $P_0 - \frac{m}{k} > 0$ $\Leftrightarrow$ $m < kP_0$.

(c) The population will be constant if $P_0 - \frac{m}{k} = 0$ $\Leftrightarrow$ $m = kP_0$. It will decline if $P_0 - \frac{m}{k} < 0$ $\Leftrightarrow$ $m > kP_0$.

(d) $P_0 = 8{,}000{,}000$, $k = \alpha - \beta = 0.016$, $m = 210{,}000$ $\Rightarrow$ $m > kP_0$ $(= 128{,}000)$, so by part (c), the population was declining.