Problem Set 31

Differential Equations Handout

14. a) $x = $ prey $y = $ predator

b) With no predators, the prey would incre[ase]
until it hit carrying capacity. With no
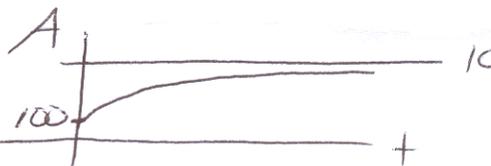prey, the predators would decrease until e[xtinct]

c) $(x,y) = \left(\frac{d}{e}, \frac{a}{c} - \frac{bd}{ce}\right)$

$$\frac{dx}{dt} = a\left(\frac{d}{e}\right) - b\left(\frac{d^2}{e}\right) - c\left(\frac{d}{e}\right)\left(\frac{a}{c} - \frac{bd}{ce}\right)$$

$$= \frac{ad - bd^2}{e} - \frac{d}{e}\left(a - bd\right)$$

$$= \frac{ad - bd^2}{e} - \frac{ad}{e} + \frac{bd^2}{e} = 0$$

$$\frac{dy}{dt} = -d\left(\frac{a}{c} - \frac{bd}{c}\right) + e\left(\frac{d}{e}\right)\left(\frac{a}{c} - \frac{bd}{c}\right)$$

$$= \frac{-ad}{c} + \frac{bd^2}{c} + d\left(\frac{a}{c} - \frac{bd}{c}\right) = 0$$
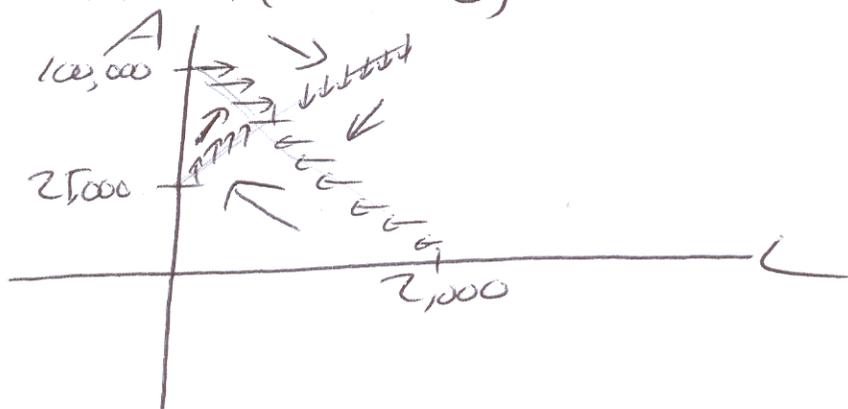
d) answers may vary

16. a) the aphids will
increase until 100,000

b) the lady bugs will
decrease until 0

c) $\frac{dA}{dt} = 0 = 100,000A - A^2 - 50AL$

$L = \frac{100,000 - A}{5}$

$\frac{dL}{dt} = 0 = -1000L - L^2 + \frac{L}{25}A$

$A = 25(L + 1000)$

Supplement

9.

a) $\frac{\partial x}{\partial t} = x - \frac{x^2}{2} - xy = 0$

$xy = x - \frac{x^2}{2}$

$y = 1 - \frac{x}{2}$

equilibrium at $\left(\frac{6}{5}, \frac{2}{5}\right)$

$\frac{\partial y}{\partial t} = -y - \frac{y^2}{2} + yx$

$yx = y + \frac{y^2}{2}$

$x = 1 + \frac{y}{2}$

$y = 2x - 2$

b)

13.

b) $\frac{\partial x}{\partial t} = 3y \qquad \frac{\partial y}{\partial t} = -3x$ (iv)

c) $\frac{\partial x}{\partial t} = 10x \qquad \frac{\partial y}{\partial t} = 10y$ (xi)

14,

d) $\frac{\partial x}{\partial t} = x - 4y$
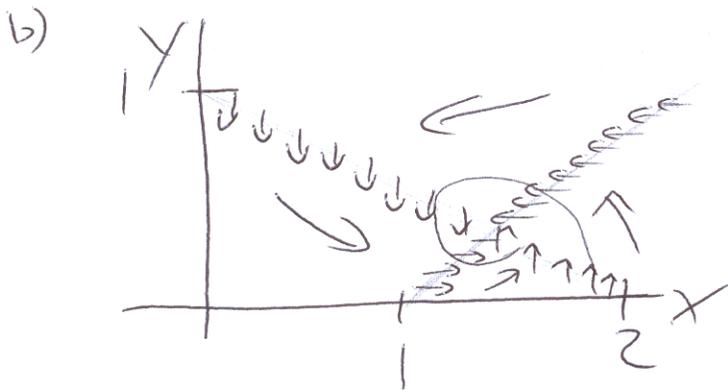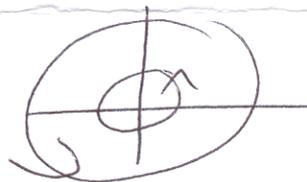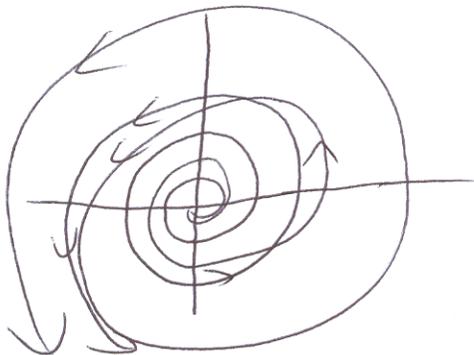
$\frac{\partial y}{\partial t} = -4x - y$

because the nullclines are at
$y = -4x$ and $x = 4y$