Problem Set 32: 19, 20, 21, 22, 23

19) a) $y'' + 6y' - 7y = 0$

$r^2 + 6r - 7 = 0$

$(r+7)(r-1) = 0$

$r = -7, 1$

general solution $Ae^{t} + Be^{-7t}$

b) $y'' + 6y' + 9y = 0$

$r^2 + 6r + 9 = 0$

$(r+3)^2 = 0$

$r = -3$

general solution $Ae^{-3t} + Bte^{-3t}$

c) $y'' + 5y' + 6y = 0$

$r^2 + 5r + 6 = 0$

$(r+3)(r+2) = 0$

$r = -3, -2$

solution $Ae^{-3t} + Be^{-2t}$

20) a) i) $y(0) = Ae^{0} + Be^{-7\cdot 0} = A + B = -2$

$y'(0) = Ae^{0} - 7Be^{-7\cdot 0} = A - 7B = 0$

then $8B = -2$ $\quad B = -\frac{1}{4}$ $\quad A = -\frac{7}{4}$

$y(t) = -\frac{7}{4}e^{t} - \frac{1}{4}e^{-7t}$

ii) $\lim_{t\to\infty} y(t) = -\infty$

b) i) $y(0) = A = -2$, $y'(t) = -3Ae^{-3t} - 3Bte^{-3t} + Be^{-3t}$

$y'(0) = -3A + B = 0$

$B = -6$

$y(t) = -2e^{-3t} - 6te^{-3t}$

ii) $\lim_{t\to\infty} y(t) = 0$

c) i) $y(0) = -2 = A + B$

$y'(t) = -3Ae^{-3t} - 2Be^{-2t}$

$y'(0) = -3A - 2B = 0$

$B = -6 \quad A = 4$

$y(t) = 4e^{-3t} - 6e^{2t}$

ii) $\lim_{t\to\infty} y(t) = 0$

21) a) $r^2+4r+3=0$ $\qquad x(0)=1$

$(r+3)(r+1)=0$ $\qquad x'(0)=2$

$r=-3,-1$

$x(t) = Ae^{-3t}+Be^{-t}$ $\qquad x'(t)=-3Ae^{-3t}-Be^{-t}$

$x(0)=1=A+B$ $\qquad x'(0)=2=-3A-B$

$3=-2A \qquad A=-3/2 \qquad B=5/2$

$x(t) = -\frac{3}{2}e^{-3t}+\frac{5}{2}e^{-t}$

b) $-\frac{3}{2}e^{-3t}+\frac{5}{2}e^{-t}=0, \quad -3e^{-3t}+5e^{-t}=0$

$5e^{-t}=3e^{-3t} \qquad \frac{e^{-t}}{e^{-3t}}=\frac{3}{5} \to e^{2t}=\frac{3}{5}$

$t=\frac{\ln\frac{3}{5}}{2} < 0$ so no the mass never crosses.

c) $x'(t)=\frac{9}{2}e^{-3t}-\frac{5}{2}e^{-t}=0$

$\to 9e^{-3t}=5e^{-t}$

$\frac{9}{5}=\frac{e^{-t}}{e^{-3t}}=e^{2t}$

$\frac{\ln 9/5}{2}=t=.29389$ seconds.

$x(.29389)=$

22) a) $C_1e^{at}+C_2e^{bt}=0 \to C_1e^{at}=-C_2e^{bt}$

$\frac{C_1}{C_2}=-\frac{e^{bt}}{e^{at}}=-e^{bt-at}=-e^{t(b-a)}$

$\ln\left(-\frac{C_1}{C_2}\right)=t(b-a)$

$\frac{\ln\left(-\frac{C_1}{C_2}\right)}{b-a}=t$ $\qquad$ only one possible value of t

b) $C_1e^{at}+C_2te^{at}=0$

$C_1+C_2t=0$

c) if it has two roots solution is like part a. One root solution is like part b. In both cases only crosses equilibrium point once.

23)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{bit} = \sum_{n=0}^{\infty} \frac{(bit)^n}{n!} = \sum_{n=0}^{\infty} \frac{i^n (bt)^n}{n!}$$

$$= 1 + \frac{i \cdot bt}{1!} - \frac{(bt)^2}{2!} - \frac{i(bt)^3}{3!} + \frac{(bt)^4}{4!} \cdots$$

$$= i \sin(bt) + \cos(bt)$$

$$e^{\pi i} = i \sin(\pi) + \cos(\pi) = -1$$