Pset 33 : ,24,26 supplement 12,13,17
for 19b, 20b see solutions to Pset 32

24) a) $y'' - 9y' = 0$ $\quad r^2 - 9r = 0 \quad r(r-9) = 0$

$r = 0, 9 \quad Ae^{0t} + Be^{9t} = A + Be^{9t}$

$y(x) = A + Be^{9x}$

b) $y'' - 9y = 0 \quad r^2 - 9 = 0 \quad (r-3)(r+3) = 0$

$r = -3, 3 \quad Ae^{-3t} + Be^{3t}$

$y(x) = Ae^{-3x} + Be^{3x}$

c) $y'' + 9y = 0 \quad r^2 + 9 = 0 \quad r^2 = -9 \quad r = \pm i3$

$y(x) = Ae^{3ix} + Be^{-3ix} = A(i\sin 3x + \cos 3x) + B(i\sin -3x + \cos -3x)$

d) $y'' - 9 = 0 \quad r^2 - 9 = 0 \quad r = \pm 3$

$y(x) = Ae^{-3x} + Be^{3x}$

e) $y'' - 2y' - y = 0 \quad r^2 - 2r - 1 = 0$

$$r = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$y(x) = Ae^{(1+\sqrt{2})x} + Be^{(1-\sqrt{2})x}$

f) $y'' - 2y' + 2y = 0 \quad r^2 - 2r + 2 = 0$

$$r = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$$

$y(x) = Ae^{(1+i)x} + Be^{(1-i)x}$

$= Ae^{x}e^{ix} + Be^{x}e^{-ix} = Ae^{x}(i\sin x + \cos x) + Be^{x}(i\sin -x + \cos -x)$

$e^{x}(C\sin x + D\cos x)$

26) a) want r to be negative for both roots,
example $(x+1)(x+2)$ so

$x^2 + 3x + 2 = 0$

$x'' + 3x' + 2x = 0$ has solution

$4e^{-t} - 3e^{-2t} = x(t)$

$\lim_{t \to \infty} x(t) = 0$

b) want r to be positive for one of the roots
example: $(x-1)(x+1)$

$x^2 - 1$

$x'' - x = 0$ has solution

$-\frac{1}{2}e^{-t} + \frac{3}{2}e^{t} = x(t)$

$\lim_{t \to \infty} x(t) = \infty$

c) need imaginary roots so it will oscillate, and need to make sure it doesn't go to zero
example $x'' + 9 = 0$ from 24c

12) In order to have periodic solutions, $x'' + bx' + cx = 0$ must have imaginary roots $a \pm bi$. Then the solution is of the form

$$y(x) = e^{ax}[C_3 \cos(bt) + C_4 \sin(bt)]$$

13) $x'' + bx' + cx = 0$ $\quad x(n) = 5$ for any integer

so periodic solution with form

$$e^{at}[C_3 \cos(bt) + C_4 \sin(bt)]$$

$$x(0) = 5 \rightarrow 5 = e^0[C_3 \cos 0 + C_4 \sin 0] = C_3$$

$$x'(t) = e^{at}[-bC_3 \sin(bt) + bC_4 \cos(bt)] + ae^{at}[C_3 \cos(bt) + C_4 \sin(bt)]$$

$$x'(0) = bC_4 + aC_3 = 0 \rightarrow C_4 = \frac{-a}{b} C_3 = \frac{-5a}{b}$$

need period to be 1 so $b = \pm 2\pi$

$$5 = e^{an}[5 \cos(\pm 2\pi n) + C_4 \sin(\pm 2\pi n)]$$

$$5 = 5e^{an} \rightarrow a = 0 \rightarrow C_4 = 0$$

solution $\quad 5 \cos(\pm 2\pi t)$

roots of polynomial are $\pm 2\pi i$ so

$$x'' + (2\pi)^2 x = 0$$

17) $e^t \sin t$ then solutions $a + bi = 1 \pm i$

$= \frac{2 \pm 2i}{2}$ which are the solutions to

$r^2 - 2r + 2$ so one solution is

$$x'' - 2x' + 2x = 0$$