Problem Set #6

6.2 21. revolve the line $y = \frac{r}{h}x$ about the x-axis.

$$V = \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$\pi \int_0^h \frac{r^2}{h^2} x^2 \, dx$$

$$= \pi \frac{r^2}{h^2}\left[\frac{1}{3}x^3\right]_0^h$$

$$= \pi \frac{r^2}{h^2}\left(\frac{1}{3}h^3\right)$$

$$= \frac{1}{3}\pi r^2 h$$

OR

revolve the line $x = -\frac{r}{h}y + r$ about the y axis

$$V = \pi \int_0^h \left(-\frac{r}{h}y + r\right)^2 dy$$

$$= \pi \int_0^h \left[\frac{r^2}{h^2}y^2 - \frac{2r^2}{h}y + r^2\right] dy$$

$$= \pi\left[\frac{r^2}{3h^2}y - \frac{r^2}{h}y^2 + r^2 y\right]_0^h$$

$$\pi\left(\frac{1}{3}r^2h - r^2h + r^2h\right)$$

$$\frac{1}{3}\pi r^2 h$$

23. V $\pi \int_{r-h}^{r} (r^2 - y^2)\, dy$

$$= \pi\left[r^2 y - \frac{y^3}{3}\right]_{r-h}^{r}$$

$$\pi\left\{\left[r^3 - \frac{r^3}{3}\right] - \left[r^2(r-h) - \frac{(r-h)^3}{3}\right]\right\}$$

$$\pi\left\{\frac{2}{3}r^3 - \frac{1}{3}(r-h)\left[3r^2 - (r-h)^2\right]\right\}$$

$$\frac{1}{3}\pi\left\{2r^3 - (r-h)\left[3r^2 - (r^2 - 2rh + h^2)\right]\right\}$$

$$\frac{1}{3}\pi\left(2r^3 - 2r^3 + 2r^2h + rh^2 + 2r^2h + 2rh - h^3\right)$$

$$= \frac{1}{3}\pi\left(3rh^2 - h^3\right)$$

$$= \pi h^2\left(r - \frac{h}{3}\right)$$

42. $y=r$ intersects $y=\sqrt{R^2-x^2}$ when

$r=\sqrt{R^2-x^2}$

$r^2=R^2-x^2$

$x^2=R^2-r^2$

$x=\pm\sqrt{R^2-r^2}$ Rotate about the x-axis.

$$V=\int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}}\pi\left[\left(\sqrt{R^2-x^2}\right)^2-r^2\right]dx$$

$$=2\pi\int_0^{\sqrt{R^2-r^2}}(R^2-x^2-r^2)\,dx$$

$$=2\pi\int_0^{\sqrt{R^2-r^2}}\left((R^2-r^2)-x^2\right)dx$$

$$=2\pi\left[(R^2-r^2)x-\frac{1}{3}x^3\right]_0^{\sqrt{R^2-r^2}}$$

$$=2\pi\left[(R^2-r^2)^{3/2}-\frac{1}{3}(R^2-r^2)^{3/2}\right]$$

$$=2\pi\cdot\frac{2}{3}(R^2-r^2)^{3/2}$$

$$=\frac{4}{3}\pi(R^2-r^2)^{3/2}$$

Integration Handout

11. a) $\pi\int_0^4 y\,dx$ b) $\pi\int_0^4(\sqrt{y}+2)^2-(2-\sqrt{y})^2\,dy$

c) $\pi\int_{-2}^{2}(4-x^2)^2\,dx$ d) $\pi\int_{-2}^{2}\left(25-(1+x^2)^2\right)dx$

12. first find total volume $V=\int_0^8\pi\left(\sqrt[3]{y}\right)^2dy$

$$=\pi\left[\frac{3}{5}y^{5/3}\right]_0^8$$

$$=\frac{96}{5}\pi$$

So half the volume is $\frac{1}{2}V=\frac{48\pi}{5}=\int_0^h\pi y^{2/3}\,dy$

$$=\pi\left[\frac{3}{5}y^{5/3}\right]_0^h$$

$$=\pi\frac{3}{5}h^{5/3}=\frac{48\pi}{5}$$

Then $16=h^{5/3}$ and

$h=16^{3/5}\approx 5.28$

Stop drinking when the height is app. 5.28 units