Solution Set: Homework 9

February 25, 2003

1 Stewart, Section 6.5

1.1 Problem 4

$25 = f(x) = kx = 0.1\,k$, so $k = 250$ N/m and $f(x) = 250x$. The work is therefore:

$$W = \int_0^{0.05} 250x\,dx = [125x^2]_0^{0.05} \approx 0.31J$$

1.2 Problem 7

The portion of rope from x to $x + \Delta x$ ft below the top of the building weighs $1/2\,\Delta x$ lb and must be lifted x_i^* ft, so its contribution to the toal work is $1/2\,x_i^*\Delta x$ ft-lb. The total work is the limit of the Riemann Sum:

$$W = \int_0^{50} 1/2\,x\,dx = [1/4\,x^2]_0^{50} = 625ft - lb.$$

1.3 Problem 8

Each part of the top 10 ft of cable is lifted a distance x_i^* equal to its distance from the top. The cable weighs $60/40 = 1.5$ lb/ft, so the work down on the ith subinterval is $3/2x_i^*\Delta x$. The remaining 30 ft of cable is lifted 10 ft, Thus

$$W = \int_0^{10} 3/2\,x\,dx + \int_{10}^{40} 3/2 \times 10\,dx = 525ft - lb$$

1.4 Problem 10

The work to lift the bucket: $4 * 80 = 320$ ft-lb. At a time t, the bucket is $x_i^* = 2t$ ft above its original 80 ft depth, but it now holds $40 - 0.2t$ lb of water. In terms of distance, the bucket holds $40 - 0.1\,x_i^*$ lb of water, so the total work is the riemann sum of this times Δx giving:

$$W = \int_0^{80} (40 - 0.1x)dx = (3200 - 320)ft - lb$$

. Adding the work to lift the bucket, you get 3200 ft-lb of work.

1.5 Problem 12

A horizontal cylindrical slice Δx of water has volume $144\pi\Delta x$ cubic feet and weighs 62.5 times that, or $9000\pi\Delta x$ lb. If the slice lies x_i^* below the edge of the pool, then the work needed to pum is $9000\pi x_i^*\Delta x$. Taking the limit, we get the integral:

$$W = \int_1^5 9000\pi x\,dx = 4500\pi(25-1) = 108,000\pi\text{ft-lb}$$

1.6 Problem 14

The work required to remove a slice is $\Delta W = 62.5\pi(5^2 - x^2)\Delta x x$ ft-lb. So the total work is:

$$W = 62.5\pi\int_0^5 x(25-x^2)\,dx = 62.5\pi(625/4) \approx 3.07\times 10^4\text{ft-lb.}$$

2 Integration Handout

2.1 Problem 22

Cutting the hemisphere into discs at equal heights x_i, we get the mass of an individual disc is:

$$\pi(25-x^2)100\Delta x$$

The force on the disk is:

$$F = ma = 9.8\pi(25-x^2)100\Delta x$$

Height the disk must move:

$$7-x$$

So the total work is the integral:

$$W = 980\pi\int_0^5 (7-x)(25-x^2)\,dx = 980\pi[175x - 12.5x^2 - 7/3x^3 + x^4/4]_0^5 = 1.315\times 10^6\text{J}$$