Problem Set #1 Math 1B Section 5.5

30. Let $u = \cos x$. Then $du = -\sin x\,dx$, so

$$\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{-du}{1+u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C$$

31. Let $u = 1 + x^2$. Then $du = 2x\,dx$, so

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx = \tan^{-1} x + \int \frac{\frac{1}{2}du}{u} = \tan^{-1} x + \tfrac{1}{2}\ln|u| + C$$

$$= \tan^{-1} x + \tfrac{1}{2}\ln|1+x^2| + C = \tan^{-1} x + \tfrac{1}{2}\ln(1+x^2) + C \quad [\text{since } 1+x^2 > 0].$$

45. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 \left(u^{3/2} + u^{1/2}\right) du = \left[\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\right]_0^1 = \tfrac{2}{5} + \tfrac{2}{3} = \tfrac{16}{15}$$

55. First write the integral as a sum of two integrals:
$\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx = \int_{-2}^{2} x\sqrt{4-x^2}\,dx + \int_{-2}^{2} 3\sqrt{4-x^2}\,dx$. The first integral is 0 by Theorem 6(b), since $f(x) = x\sqrt{4-x^2}$ is an odd function and we are integrating from $x = -2$ to $x = 2$. The second integral we interpret as three times the area of a semicircle with radius 2, so the original integral is equal to $0 + 3 \cdot \frac{1}{2}(\pi \cdot 2^2) = 6\pi$.

62. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 x f(x^2)\,dx = \int_0^9 f(u)\left(\frac{1}{2}du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$.

Int. Handout

(2) a) $\int \sin^2 x \cos^3 x\,dx$

$\sin^2 x \cos^2 x \cos x\,dx$

$= \int \sin^2 x (1 - \sin^2 x)\cos x\,dx$

$\int \sin^2 x \cos x\,dx - \int \sin^4 x \cos x\,dx$

$$\boxed{= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}$$

b) $\int \sin^5 x \cos^4 x\,dx$

$= \int \sin x \sin^4 x \cos^4 x\,dx$

$= \int \sin x (1 - \cos^2 x)^2 \cos^4 x\,dx$

$= \int \sin x (\cos^4 x - 2\cos^6 x + \cos^8 x)\,dx$

$= \int \cos^4 x \sin x\,dx - 2\int \cos^6 x \sin x\,dx + \int \cos^8 x \sin x\,dx$

$= -\frac{\cos^5 x}{5} + \frac{2\cos^7 x}{7} - \frac{\cos^9 x}{9} + C$