Solution Set: Homework 10

February 28, 2003

1 Stewart, Section 6.5

1.1 Problem 13

A rectangular slice of water Δx thick and lying x feet above the bottom has width x and volume $8x\Delta x$. It weighs $(9800)(8x\Delta x)$ and must be lifted $5 - x$ meters by the pump, so the work is:

$$W = \int_0^3 (9.8 \times 10^3)(5 - x)8x\, dx = 1058.4 \times 10^3 \mathrm{J}$$

2 Integration Handout

2.1 Problem 26

For all values of $p \neq 1$, we have:

$$\int_1^\infty \frac{1}{x^p}\, dx = \lim_{a\to\infty} \left[\frac{x^{1-p}}{(1-p)}\right]_1^a = \lim_{a\to\infty} \frac{a^{1-p}}{(1-p)} - \frac{1}{(1-p)}$$

This converges whenever $1-p < 0$, in which case the integral has value $1/(1-p)$. For $1 - p > 0$, this integral diverges. When $1 = p$, the integral becomes:

$$\lim_{a\to\infty} [\log x]_1^a = \lim_{a\to\infty} \log a - \log 1$$

Which diverges.

3 Stewart, Section 5.10

3.1 Problem 2

(a) $1/(2x - 1)$ defined and continuous on $[1, 2]$, so the integral is proper. (b) $1/(2x - 1)$ discontinuous at $x = 1/2$, so the integral is improper (Type II). (c) The integral has an infinite interval of integration so it is improper (Type I). (d) $\log x - 1$ is discontinuous at $x = 1$ so the integral is improper (Type II).

3.2 Problem 6

$$\int_0^\infty dx/(x+3)^{3/2} = \lim_{t\to\infty}\int_0^t dx/(x+3)^{3/2} = \lim_{t\to\infty}[-2/Sqrtx+3]_2^t = 2/Sqrt5$$

(Convergent)

3.3 Problem 12

$$\int_{-\infty}^{\infty}(2-v^4)\,dx = \int_0^\infty (2-v^4)\,dx + \int_{-\infty}^0 (2-v^4)\,dx$$

The first integral diverges because it is:

$$\lim_{t\to\infty}[2v - v^5/5]_0^\infty$$

So the whole integral diverges.

3.4 Problem 14

Split the integral up into two regions from 0 to infinity and from -infinity to 0 and observe that the latter diverges:

$$\int_{-\infty}^0 x^2 e^{-x^3} = -1/3 + 1/3(\lim_{t\to-\infty} e^{-t^3})$$

Which diverges.

3.5 Problem 20

The integrand is even, so we can double the integral from 0 to ∞:

$$\int_{-\infty}^{\infty} 1/(r^2+4)dr = 2\int_0^\infty 1/(r^2+4)dr = 2\lim_{t\to-\infty}[1/2\arctan r/2]_0^t = \pi/2$$

Convergent.

3.6 Problem 51

(a) Split the integral in two at 0:

$$\int_{-\infty}^0 xdx + \int_0^\infty xdx$$

And observe that the second diverges as it is:

$$\lim_{t\to\infty}[x^2/2]_0^t$$

(b) The integral is always zero since the integrand is an odd function. So the limit is 0. So therefore, $\int_{-\infty}^{\infty} xdx \neq \lim_{t\to\infty}\int_{-t}^{t} xdx$.