Problem Set 29

Differential Equations HandOut

10. a) i) $y(0)=0$ means the solution is the equilibrium line. It is a horizontal line at $y=0$.

ii) $y(0)=.01$ means the solution is an exponential curve that is always positive and increasing

iii) $y(0)=-.01$ means that the solution is an exponential curve that is always negative and decreasing

b) unstable equilibrium (or semi-stable)

c) $\frac{dy}{dx}=y^2 \rightarrow \frac{dy}{y^2}=1dx \rightarrow \int \frac{dy}{y^2}=\int dx$

$\rightarrow \frac{-1}{y}=x+C \rightarrow y=\frac{-1}{x+C} \rightarrow 1=\frac{-1}{0+C} \rightarrow C=-1$

$$y=\frac{-1}{x-1}$$

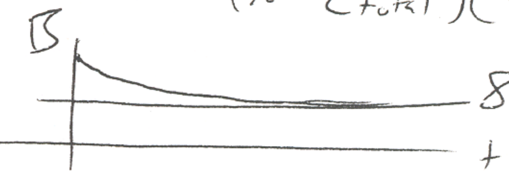
d) as x goes to 1, y is an asymptote

11. a) $\frac{dW}{dt}=\left(.2\frac{L\text{ white}}{L\text{ total}}\right)\left(2\frac{L\text{ total}}{\text{hour}}\right)-\left(\frac{W}{10}\frac{L\text{ white}}{L\text{ total}}\right)\left(2\frac{L\text{ total}}{\text{hour}}\right)$

$W(0)=0$

b) $\frac{dB}{dt}=\left(.8\frac{L\text{ blue}}{L\text{ total}}\right)\left(2\frac{L\text{ total}}{\text{hour}}\right)-\left(\frac{B}{10}\frac{L\text{ blue}}{L\text{ total}}\right)\left(2\frac{L\text{ total}}{\text{hour}}\right)$

$B(0)=10$



12. $f(x)$ can't ever be both increasing and concave up because then both $y'>0$ and $y''>0$. So then $y''+y'>0$, but $y''+y'=-x^2<0$!

First Order Linear Handout

1. $y' - 4xy = x \qquad v(x) = e^{\int -4x\,dx} = e^{-2x^2}$

$$e^{-2x^2} y = \int e^{-2x^2} x\,dx = -\frac{1}{4} e^{-2x^2} + C$$

$$y = -\frac{1}{4} + Ce^{2x^2}$$

2. $y' - \frac{3}{x} y = x - x^3 \qquad v(x) = e^{\int -\frac{3}{x}\,dx} = e^{-3\ln x} = x^{-3}$

$$x^{-3} y = \int x^{-3}(x - x^3)\,dx = \int \frac{1}{x^2} - 1\,dx = \frac{-1}{x} - x + C$$

$$y = -x^2 - x^4 + Cx^3$$

3. $y' + y = e^x,\ y(0) = 6 \qquad v(x) = e^{\int dx} = e^x$

$$e^x y = \int e^x \cdot e^x\,dx = \int e^{2x}\,dx = \frac{1}{2} e^{2x} + C$$

$$y = \frac{1}{2} e^x + \frac{C}{e^x} \rightarrow 6 = \frac{1}{2} e^0 + \frac{C}{e^0} = \frac{1}{2} + C \Rightarrow C = \frac{11}{2}$$

$$y = \frac{1}{2} e^x + \frac{11}{2e^x}$$

Section 7.3

34. a) $\frac{dx}{dt} = \frac{10 - x}{10}(.05) = .005(10 - x)$

b) $\frac{dx}{dt} = .005(10 - x) \rightarrow \frac{dx}{10 - x} = .005\,dt \rightarrow \int \frac{dx}{x - 10} = \int -.005\,dt$

$$\rightarrow \ln(10 - x) = -.005t + C \rightarrow 10 - x = Ce^{-.005t} \rightarrow$$

$$x = 10 - Ce^{-.005t} \rightarrow 0 = 10 = Ce^0 \rightarrow C = 10$$

$$x = 10 - 10e^{-.005t}$$

c) $x = .9(10) = 9 = 10 - 10e^{-.005t} \rightarrow e^{-.005t} = .1$

$$\rightarrow -.005t = -\ln 10 \rightarrow t = 200 \ln 10 \approx 460.5 \text{ days}$$

36. a) $\frac{dy}{dt} = \left(.05 \frac{kg}{L}\right)\left(5 \frac{L}{min}\right) + \left(.04 \frac{kg}{L}\right)\left(10 \frac{L}{min}\right) - \left(\frac{y}{100} \frac{kg}{L}\right)\left(15 \frac{L}{min}\right)$

$$= \frac{130 - 3y}{200} \frac{kg}{min} \qquad \text{Then } y = \frac{130}{3}\left(1 - e^{-3t/200}\right) \text{ kg}$$

b) $y(1) = \frac{130}{3}\left(1 - e^{-3/200}\right) \approx 25.7 \text{ kg}$