Problem Set 31

Differential Equations Handout

14. a) x = prey y = predator

b) With no predators, the prey would incre until it hit carrying capacity. With no prey, the predators would decrease until e

c) $(x,y) = \left(\frac{d}{e}, \frac{a}{c} - \frac{bd}{ce}\right)$

$$\frac{dx}{dt} = a\left(\frac{d}{e}\right) - b\left(\frac{d^2}{e}\right) - c\left(\frac{d}{e}\right)\left(\frac{a}{c} - \frac{bd}{ce}\right)$$

$$= \frac{ad - bd^2}{e} - \frac{d}{e}(a - bd)$$

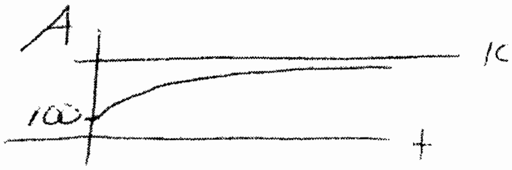
$$= \frac{ad - bd^2}{e} - \frac{ad}{e} + \frac{bd^2}{e} = 0$$

$$\frac{dy}{dt} = -d\left(\frac{a}{c} - \frac{bd}{c}\right) + e\left(\frac{d}{e}\right)\left(\frac{a}{c} - \frac{bd}{c}\right)$$

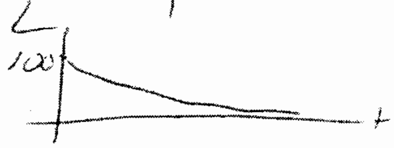
$$= \frac{-ad}{c} + \frac{bd^2}{c} + d\left(\frac{a}{c} - \frac{bd}{c}\right) = 0$$

d) answers may vary

16. a) the aphids will increase until 100,000



b) the lady bugs will decrease until 0

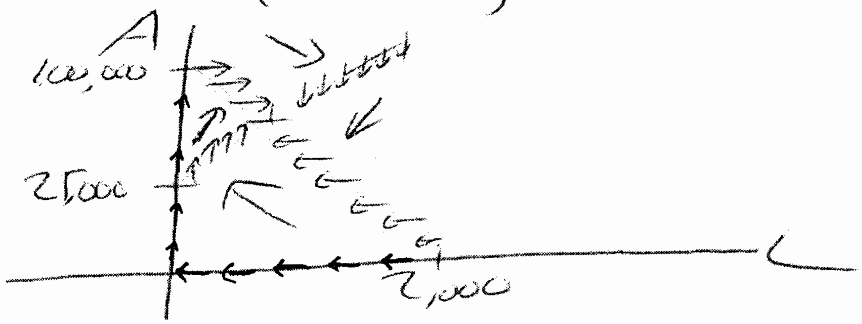


c) $\frac{dA}{dt} = 0 = 100{,}000A - A^2 - 50AL$

$$L = \frac{100{,}000 - A}{5}$$

$$\frac{dL}{dt} = 0 = -1000L - L^2 + \frac{L}{25}A$$

$$A = 25(L + 1000)$$



Supplement

7\. a) $\frac{\partial x}{\partial t} = x - \frac{x^2}{2} - xy = 0$

$xy = x - \frac{x^2}{2}$

$y = 1 - \frac{x}{2}$

equilibrium at $\left(\frac{6}{5}, \frac{2}{5}\right)$

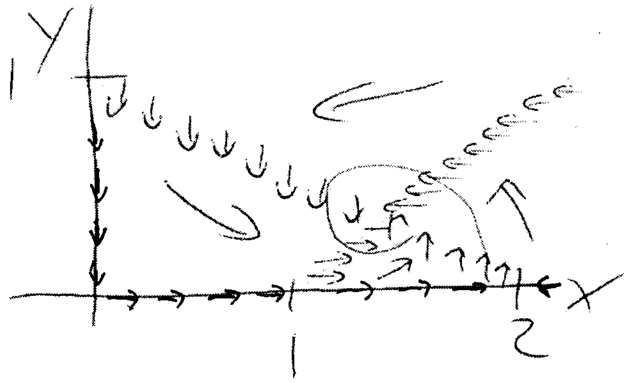
$\frac{\partial y}{\partial t} = -y - \frac{y^2}{2} + yx$

$yx = \cancel{y} + \frac{y^2}{2}$

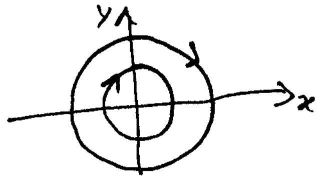
$x = 1 + \frac{y}{2}$

$y = 2x - 2$

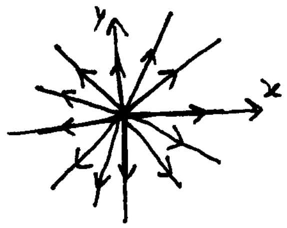
b)



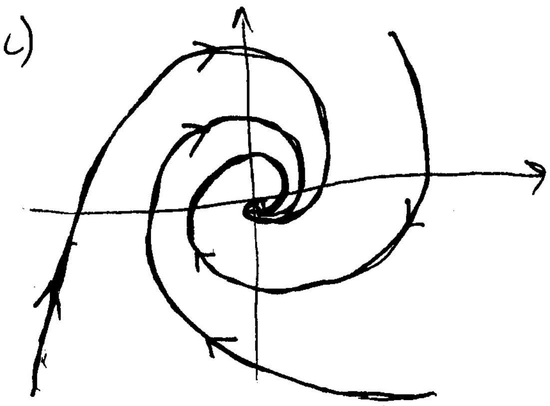
13\. b) $\frac{dx}{dt} = 3y \quad \frac{dy}{dt} = -3x$ (ix)



c) $\frac{dx}{dt} = 10x \quad \frac{dy}{dt} = 10y$ (vii)



14\. c)



$\frac{dx}{dt} = -x + 4y$

$\frac{dy}{dt} = -4x - y$

Nullclines

$y = \frac{1}{4}x$

$y = -4x$