

19) a)  $y'' + 6y' - 7y = 0$   
 $r^2 + 6r - 7 = 0$   
 $(r+7)(r-1) = 0$   
 $r = -7, 1$

general solution  $Ae^t + Be^{-7t}$

b)  $y'' + 6y' + 9y = 0$   
 $r^2 + 6r + 9 = 0$   
 $(r+3)^2 = 0$   
 $r = -3$

general solution  $Ae^{-3t} + Bte^{-3t}$

c)  $y'' + 5y' + 6y = 0$   
 $r^2 + 5r + 6 = 0$   
 $(r+3)(r+2) = 0$   
 $r = -3, -2$

solution  $Ae^{-3t} + Be^{-2t}$

20) a) i)  $y(0) = Ae^0 + Be^{-7 \cdot 0} = A + B = -2$

$y'(0) = Ae^0 - 7Be^{-7 \cdot 0} = A - 7B = 0$

then  $8B = -2$   $B = -\frac{1}{4}$   $A = -\frac{7}{4}$

ii)  $y(t) = -\frac{7}{4}e^t - \frac{1}{4}e^{-7t}$   
 $\lim_{t \rightarrow \infty} y(t) = -\infty$

b) i)  $y(0) = A = -2$ ,  $y'(t) = -3Ae^{-3t} - 3Bte^{-3t} + Be^{-3t}$   
 $y'(0) = -3A + B = 0$   
 $B = -6$

ii)  $y(t) = -2e^{-3t} - 6te^{-3t}$   
 $\lim_{t \rightarrow \infty} y(t) = 0$

c) i)  $y(0) = -2 = A + B$   
 $y'(t) = -3Ae^{-3t} - 2Be^{-2t}$   
 $y'(0) = -3A - 2B = 0$

$B = -6$   $A = 4$

ii)  $y(t) = 4e^{-3t} - 6e^{-2t}$   
 $\lim_{t \rightarrow \infty} y(t) = 0$

$$21) \quad a) \quad r^2 + 4r + 3 = 0$$

$$(r+3)(r+1) = 0$$

$$x(0) = 1$$

$$x'(0) = 2$$

$$r = -3, -1$$

$$x(t) = Ae^{-3t} + Be^{-t}$$

$$x'(t) = -3Ae^{-3t} - Be^{-t}$$

$$x(0) = 1 = A + B$$

$$x'(0) = 2 = -3A - B$$

$$3 = -2A$$

$$A = -3/2$$

$$B = 5/2$$

$$x(t) = -\frac{3}{2}e^{-3t} + \frac{5}{2}e^{-t}$$

$$b) \quad -\frac{3}{2}e^{-3t} + \frac{5}{2}e^{-t} = 0, \quad -3e^{-3t} + 5e^{-t} = 0$$

$$5e^{-t} = 3e^{-3t}$$

$$\frac{e^{-t}}{e^{-3t}} = \frac{3}{5} \rightarrow e^{2t} = \frac{3}{5}$$

$$t = \frac{\ln \frac{3}{5}}{2} < 0 \quad \text{so no the mass never crosses.}$$

$$c) \quad x'(t) = \frac{9}{2}e^{-3t} - \frac{5}{2}e^{-t} = 0$$

$$\rightarrow 9e^{-3t} = 5e^{-t}$$

$$\frac{9}{5} = \frac{e^{-t}}{e^{3t}} = e^{2t}$$

$$\frac{\ln 9/5}{2} = t = .29389 \text{ seconds}$$

$$x(.29389) =$$