Pset #33

DEH 19 b) $y'' + 6y' + 9y = 0$ $\quad r^2 + 6r + 9 = 0 \Leftrightarrow (r+3)^2 = 0 \Leftrightarrow r = -3$

General solution $Ae^{-3t} + Bte^{-3t}$

20 b) i) $y(0) = A = -2$, $y'(t) = -3Ae^{-3t} - 3Bte^{-3t} + Be^{-3t}$

$y'(0) = -3A + B = 0$

$B = -6$ $\quad y(t) = -2e^{-3t} - 6te^{-3t}$

ii) $\lim_{t \to \infty} y(t) = 0$

22) a) $C_1 e^{at} + C_2 e^{bt} = 0 \rightarrow C_1 e^{at} = -C_2 e^{bt}$

$$\frac{C_1}{C_2} = -\frac{e^{bt}}{e^{at}} = -e^{bt-at} = -e^{t(b-a)}$$

$$\ln\left(-\frac{C_1}{C_2}\right) = t(b-a)$$

$$\frac{\ln\left(-\frac{C_1}{C_2}\right)}{b-a} = t$$

only one possible value of t

b) $C_1 e^{at} + C_2 t e^{at} = 0$

c) if it has two roots solution is like part a. One root solution is like part b. In both cases only crosses equilibrium point once.

23) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$e^{bit} = \sum_{n=0}^{\infty} \frac{(bit)^n}{n!} = \sum_{n=0}^{\infty} \frac{i^n (bt)^n}{n!}$$

$$= 1 + \frac{i \cdot bt}{1!} - \frac{(bt)^2}{2!} - \frac{i(bt)^3}{3!} + \frac{(bt)^4}{4!} \cdots$$

$$= i \sin(bt) + \cos(bt)$$

$e^{\pi i} = i \sin(\pi) + \cos(\pi) = -1$

24) a) $y'' - 9y' = 0$ $\quad r^2 - 9r = 0$ $\quad r(r-9) = 0$

$r = 0, 9$ $\quad Ae^{0t} + Be^{9t} = A + Be^{9t}$

$y(x) = A + Be^{9x}$

b) $y'' - 9y = 0$ $\quad r^2 - 9 = 0$ $\quad (r-3)(r+3) = 0$

$r = -3, 3$ $\quad Ae^{-3t} + Be^{3t}$

$y(x) = Ae^{-3x} + Be^{3x}$

d) $y'' - 9 = 0$ → integrate $y' = 9t + C_1$

→ integrate again $y = \frac{9}{2}t^2 + C_1 t + C_2$